Faster Algorithms for Approximating Combinatorial and Geometric Data

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2025-07-16

Université Sorbonne Paris Nord

Introduction

$X \longleftarrow$ ground set

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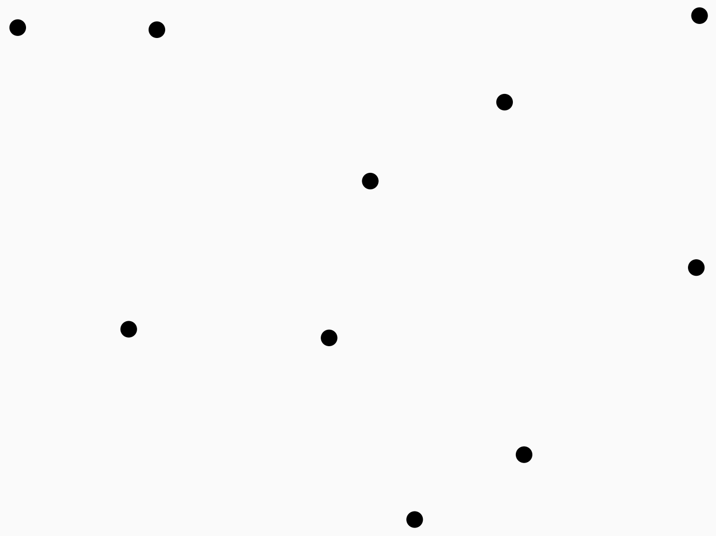

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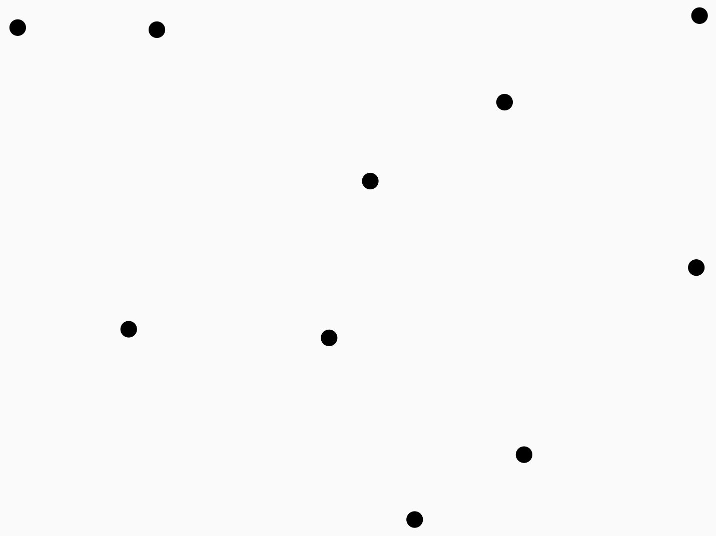

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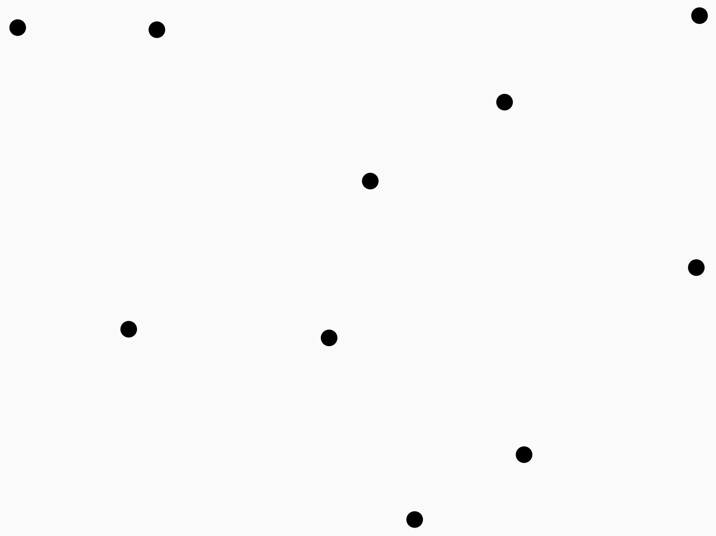
$\mathcal{F} \longleftarrow$ collection of subsets of X



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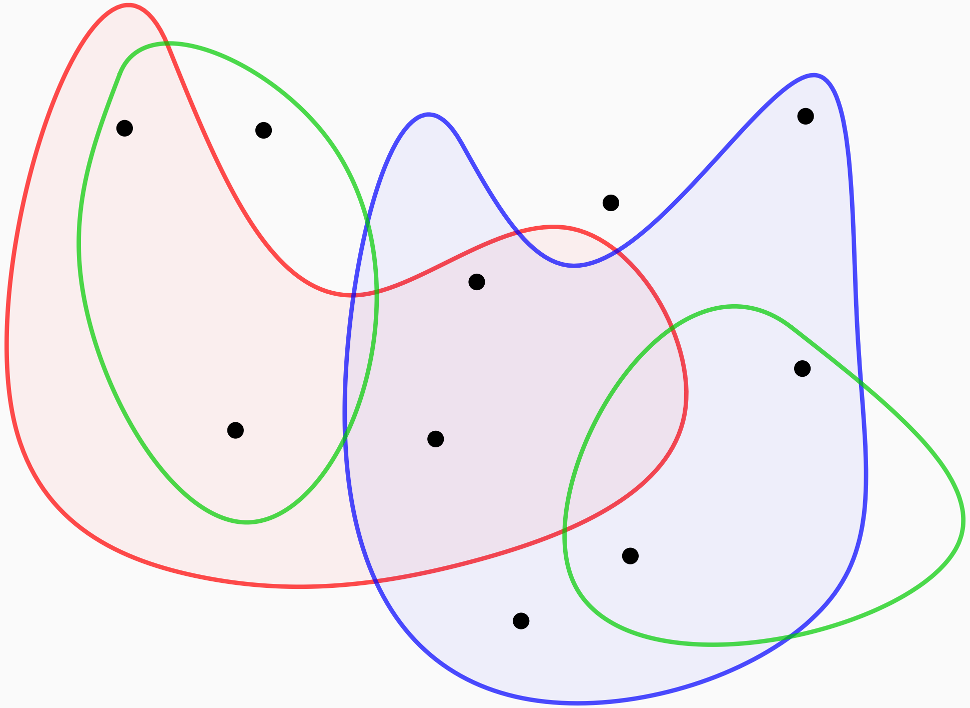

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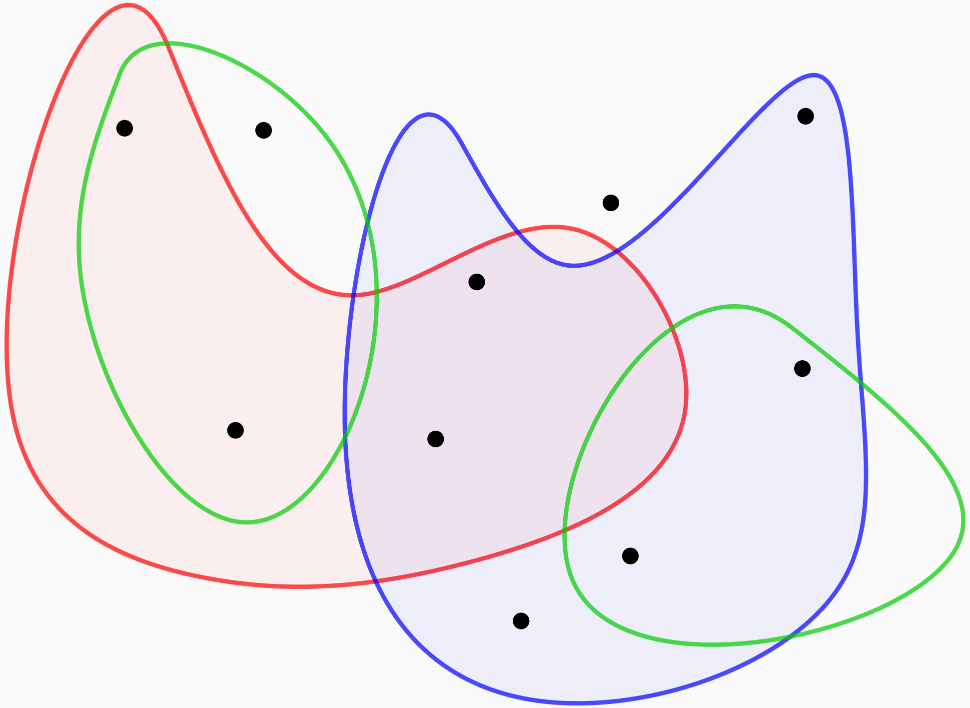

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$\Rightarrow$ combinatorially $m \leq 2^n$

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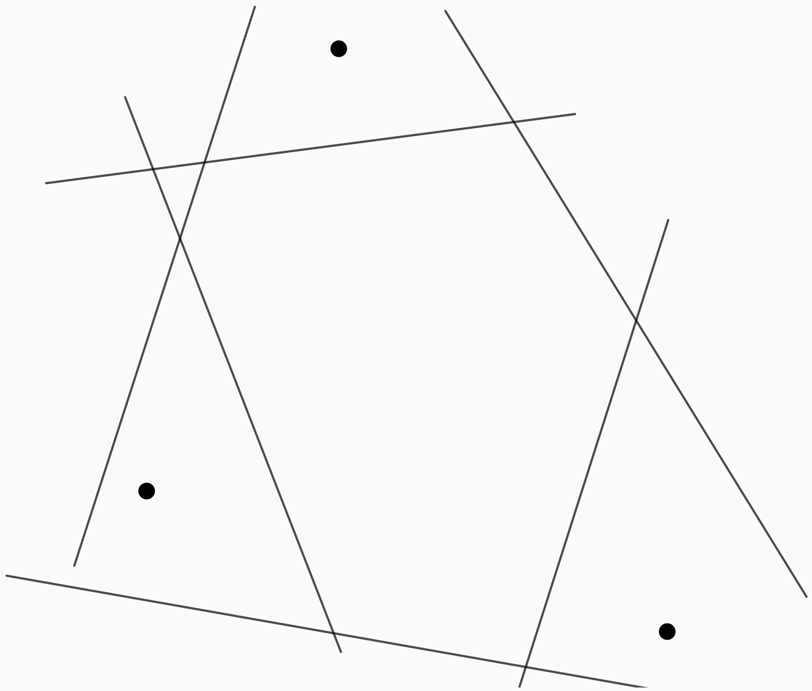

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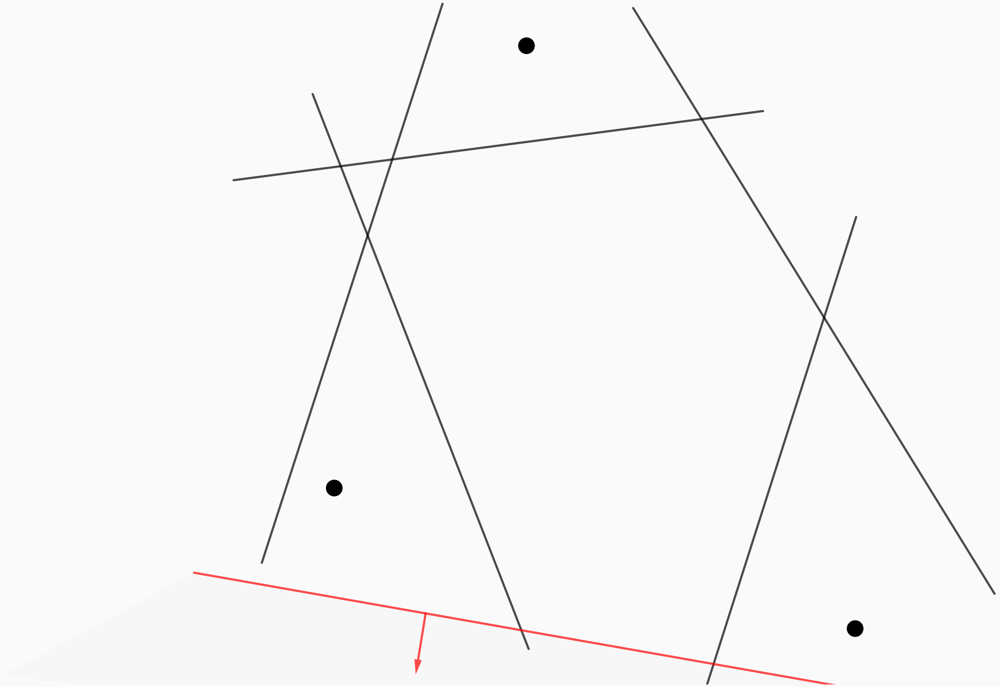

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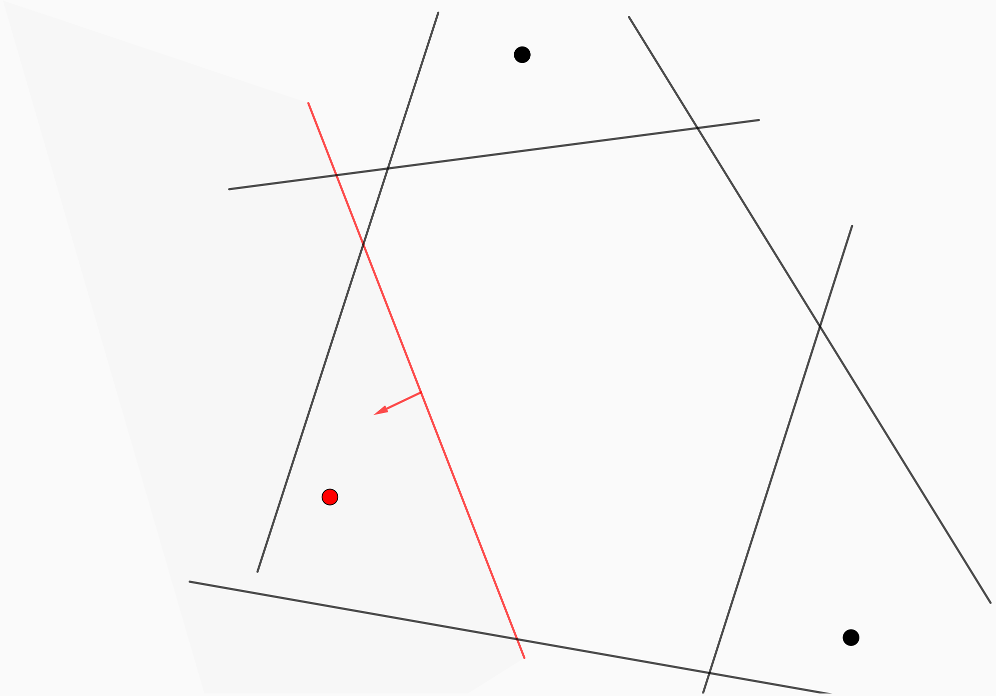

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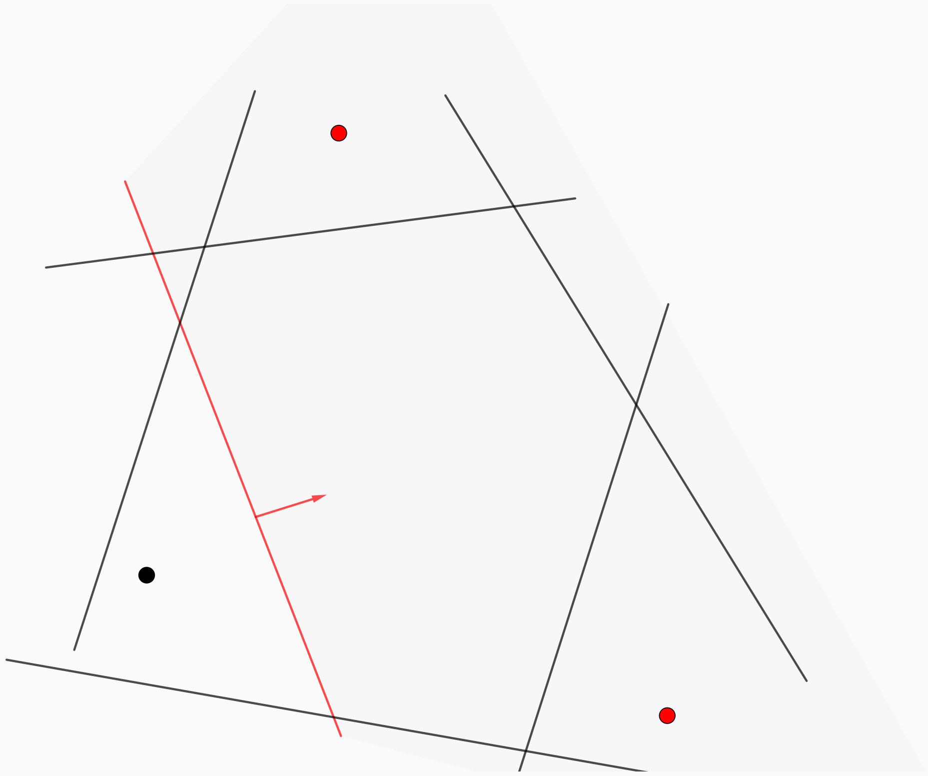

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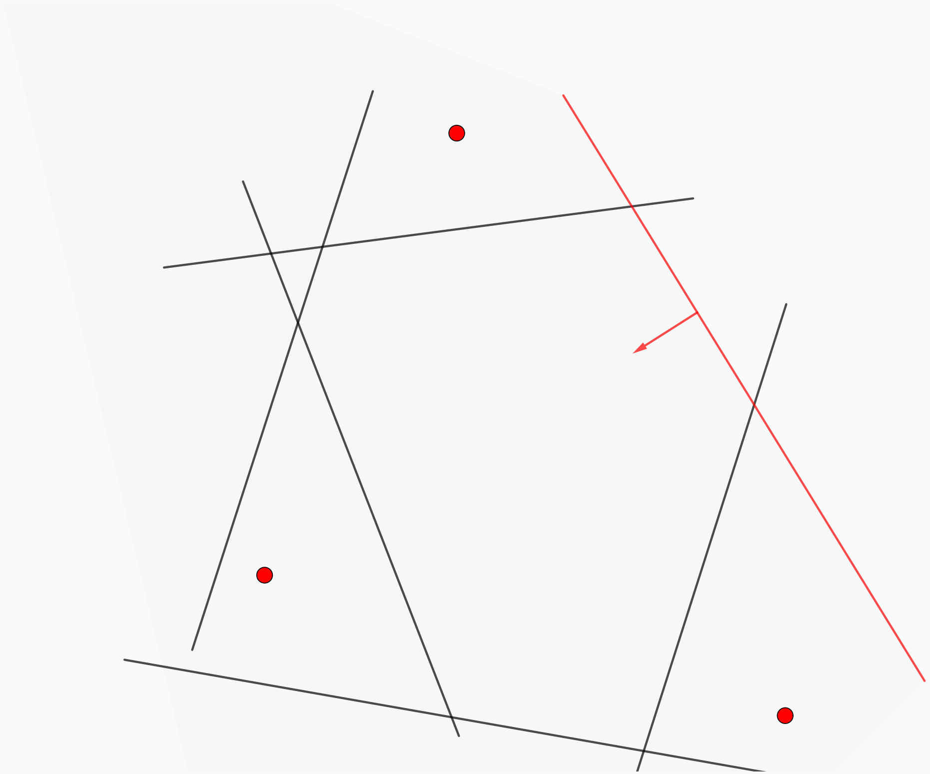

VC-dimension

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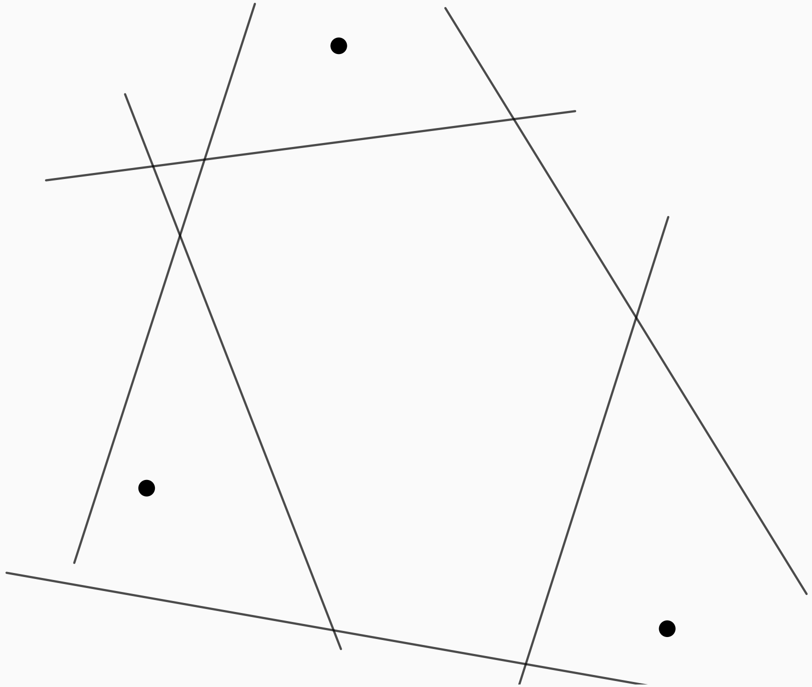

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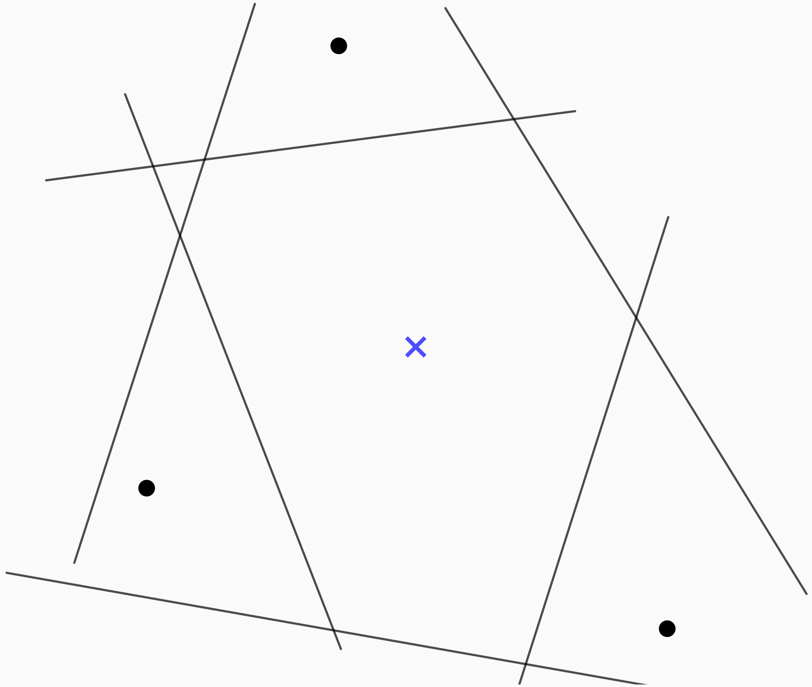

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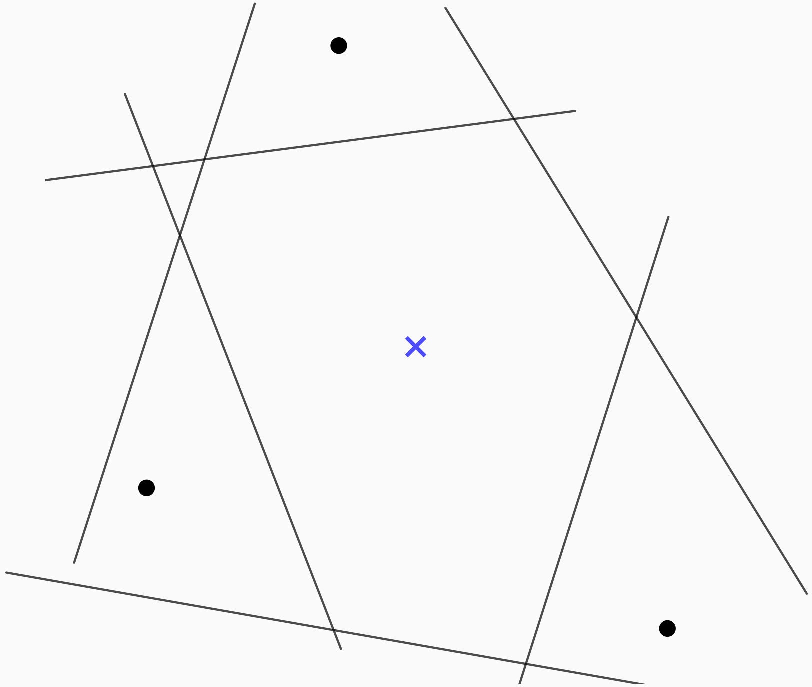

VC-dimension: $\sup_{d \in [1,n]} \exists Y \subseteq X, |Y| = d \text{ s.t. } |\{F \cap Y : F \in \mathcal{F}\}| = 2^d$

Combinatorial discrepancy

2-coloring: $\chi : X \to \{-1, +1\}$

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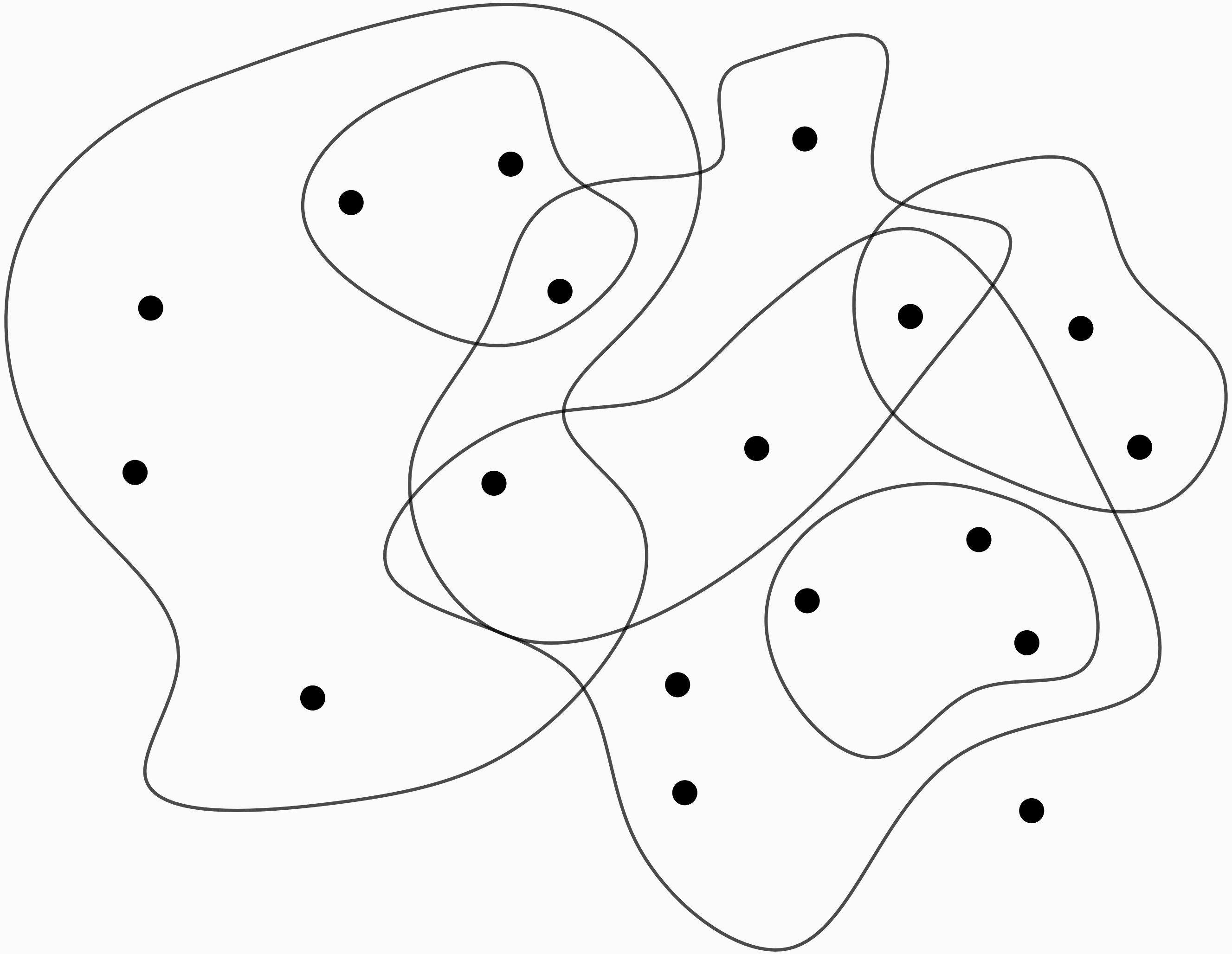

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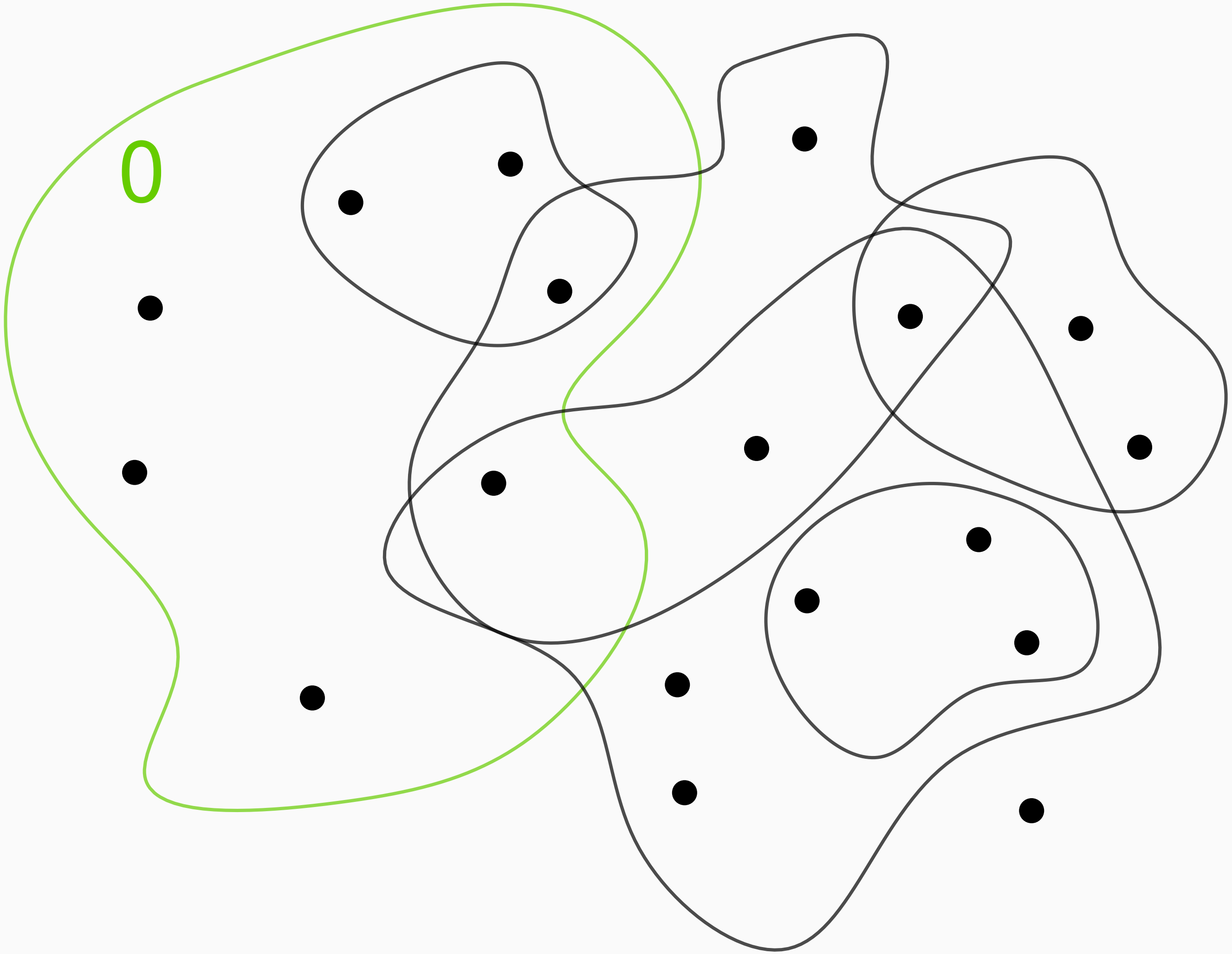

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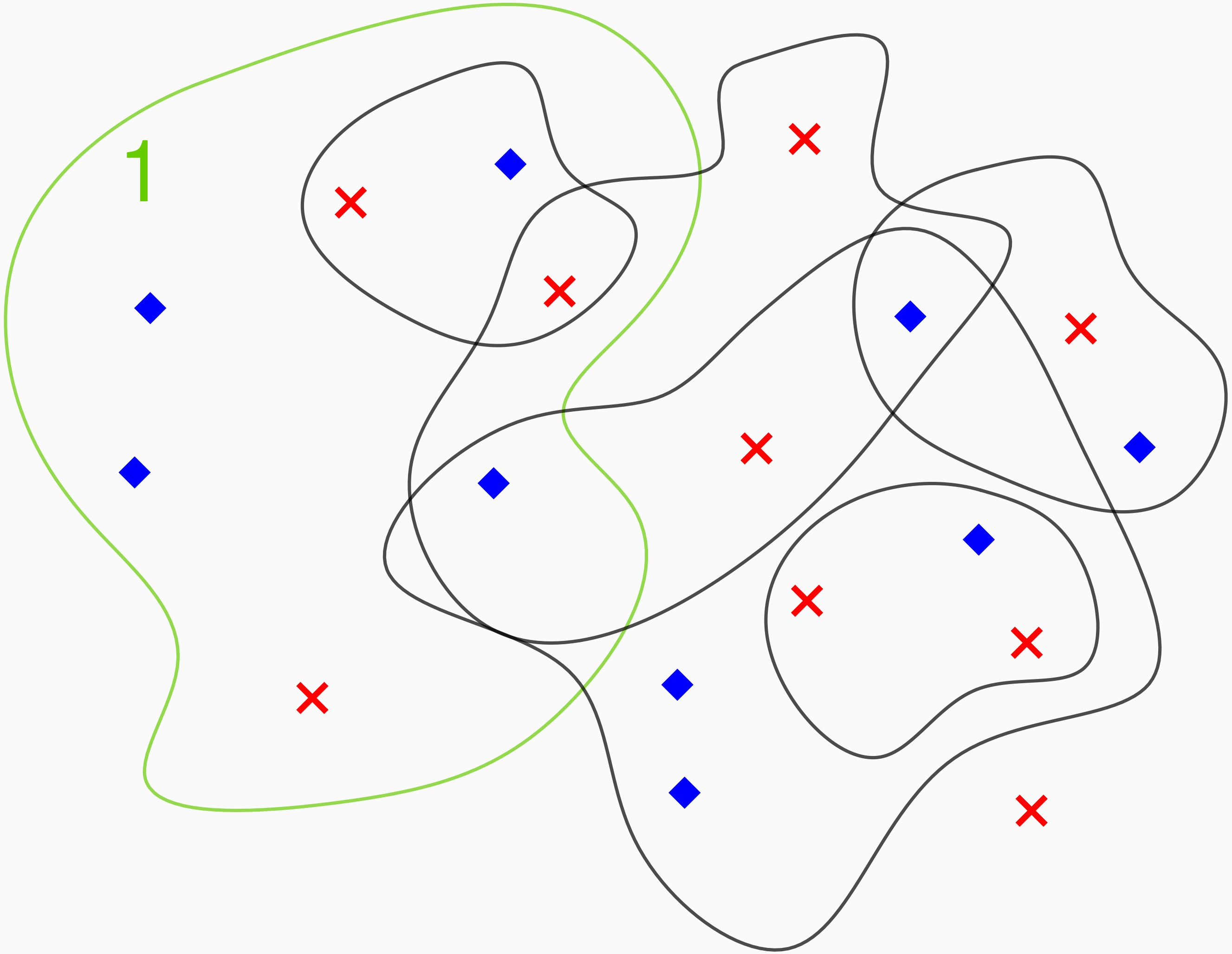

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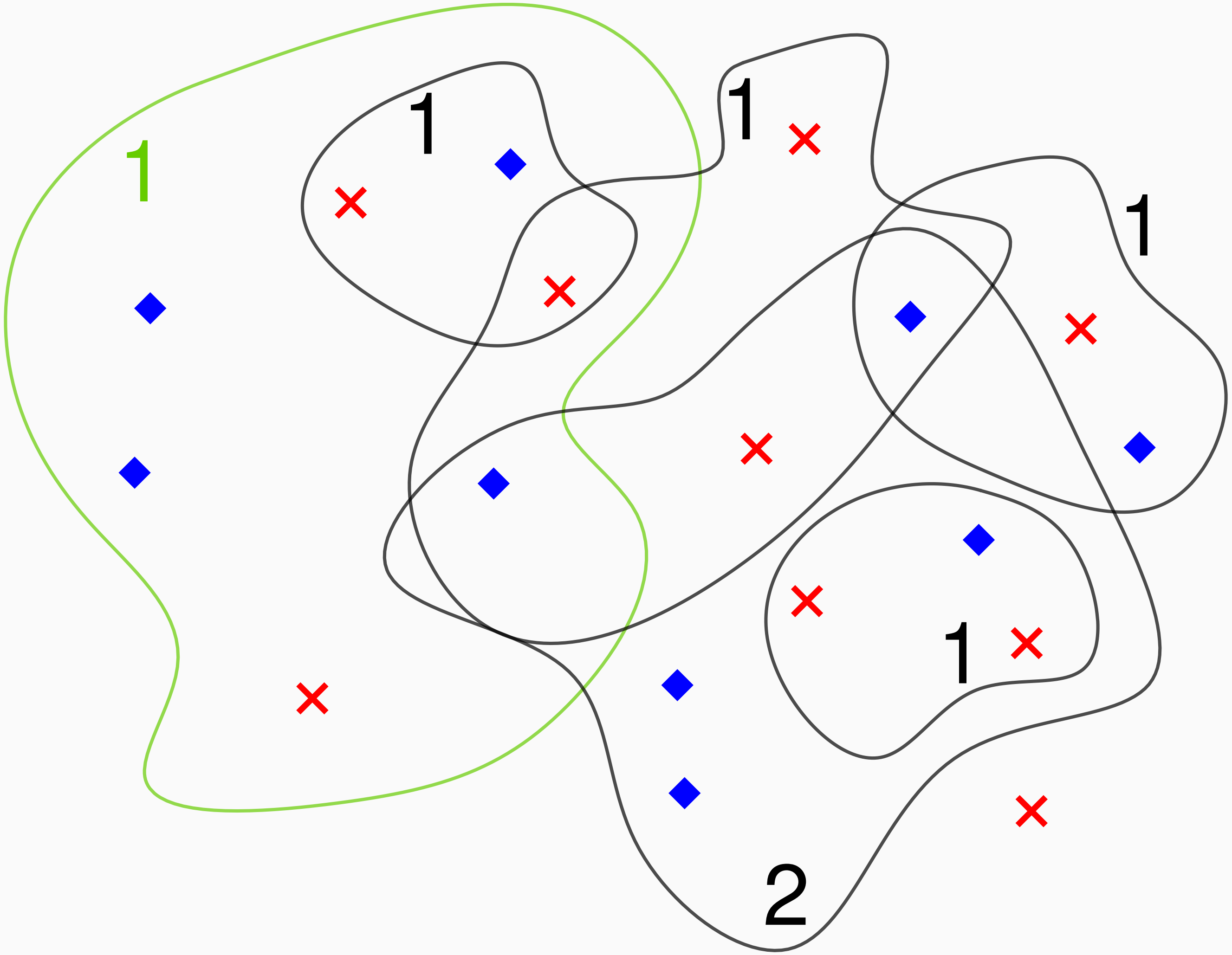

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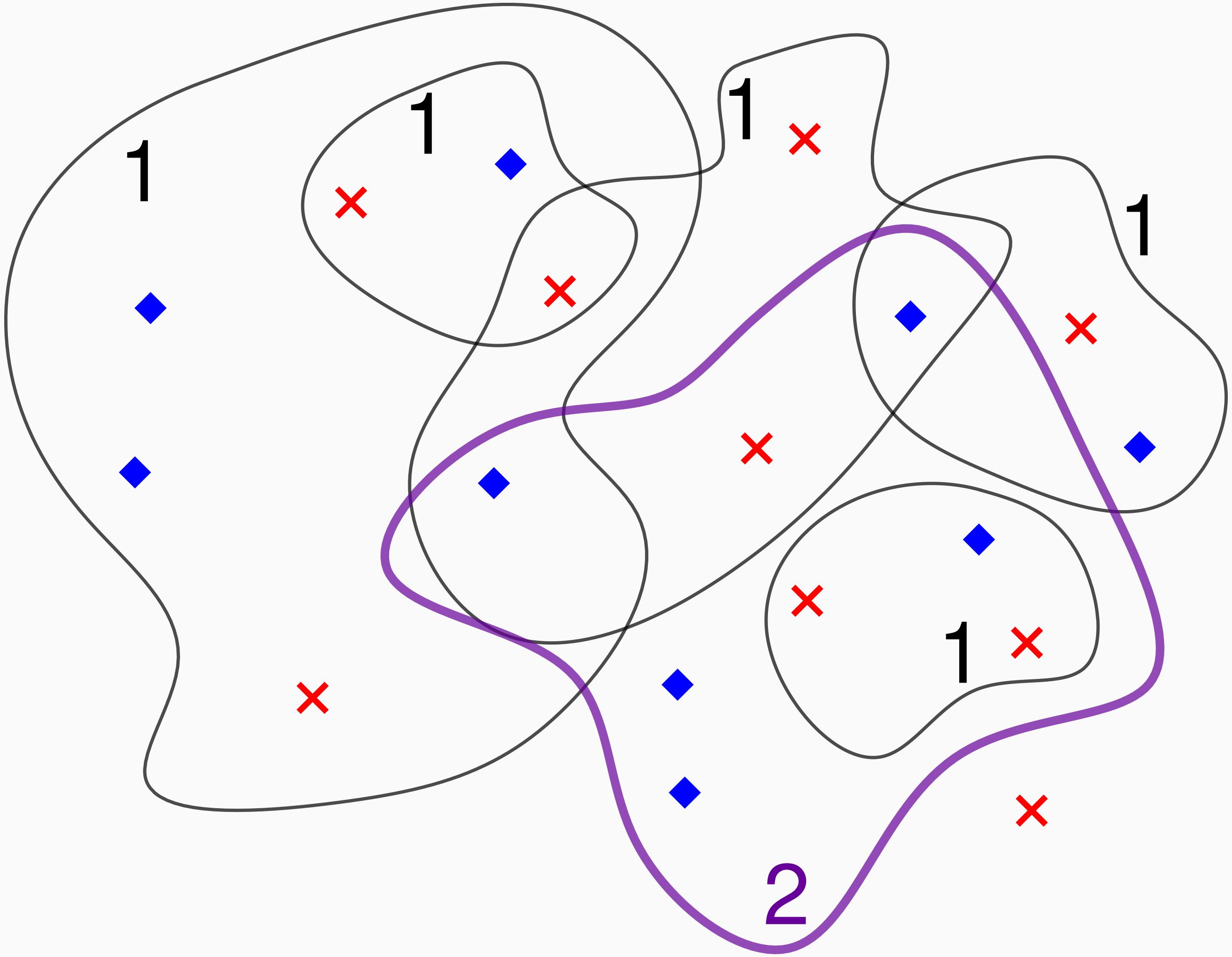

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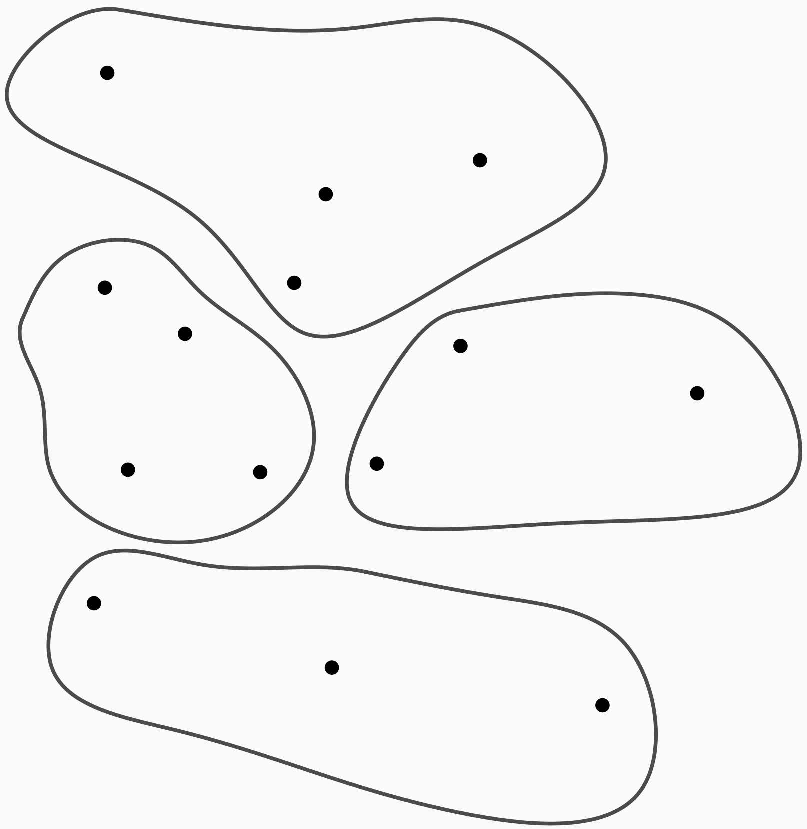
Goal: Compute a small discrepancy coloring

Discrepancy and approximations



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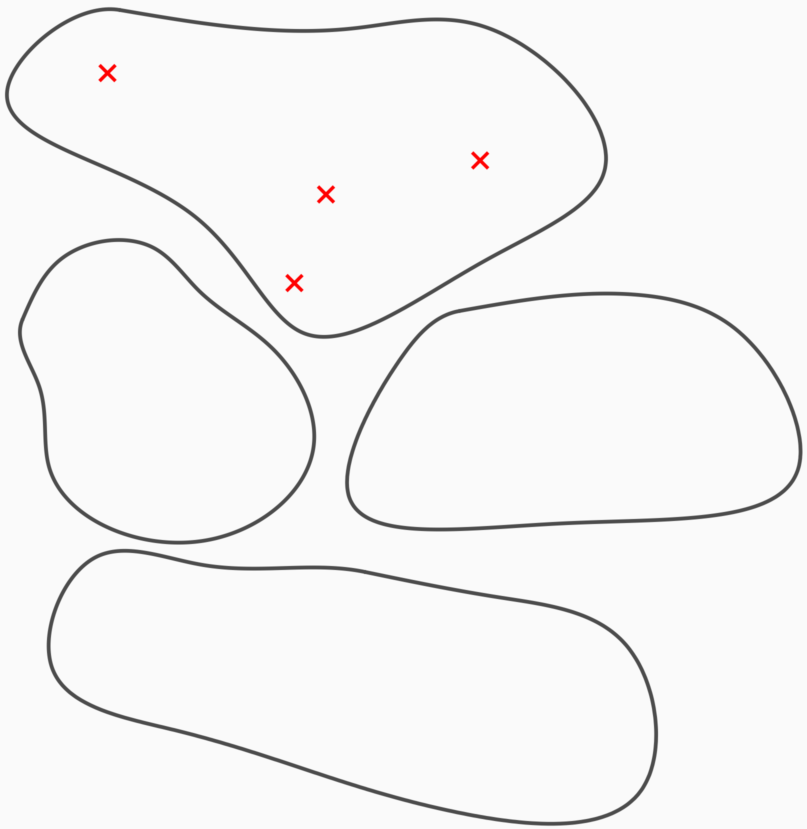

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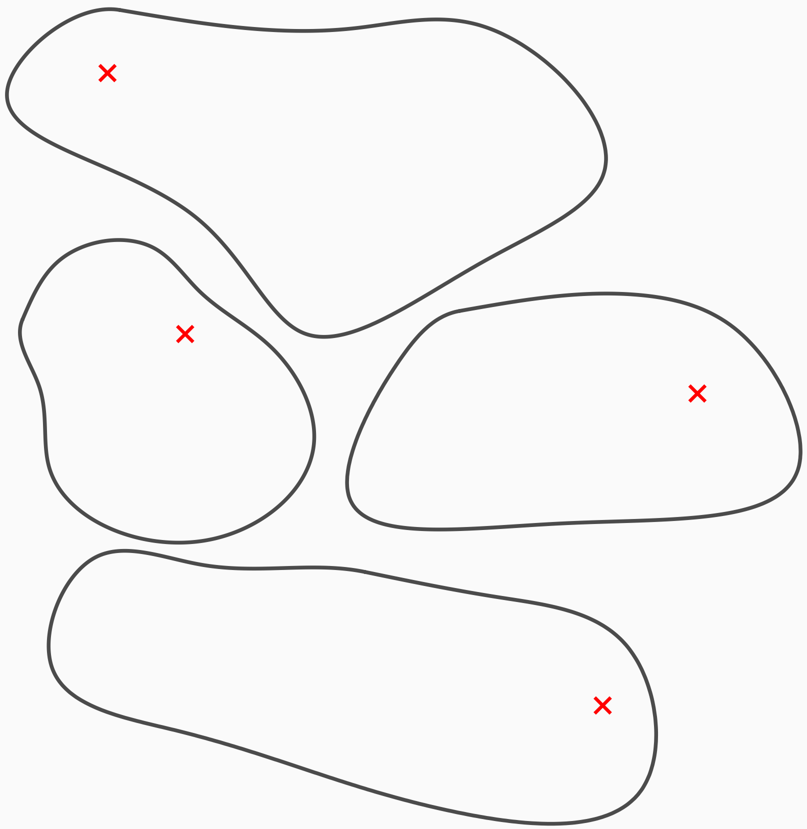
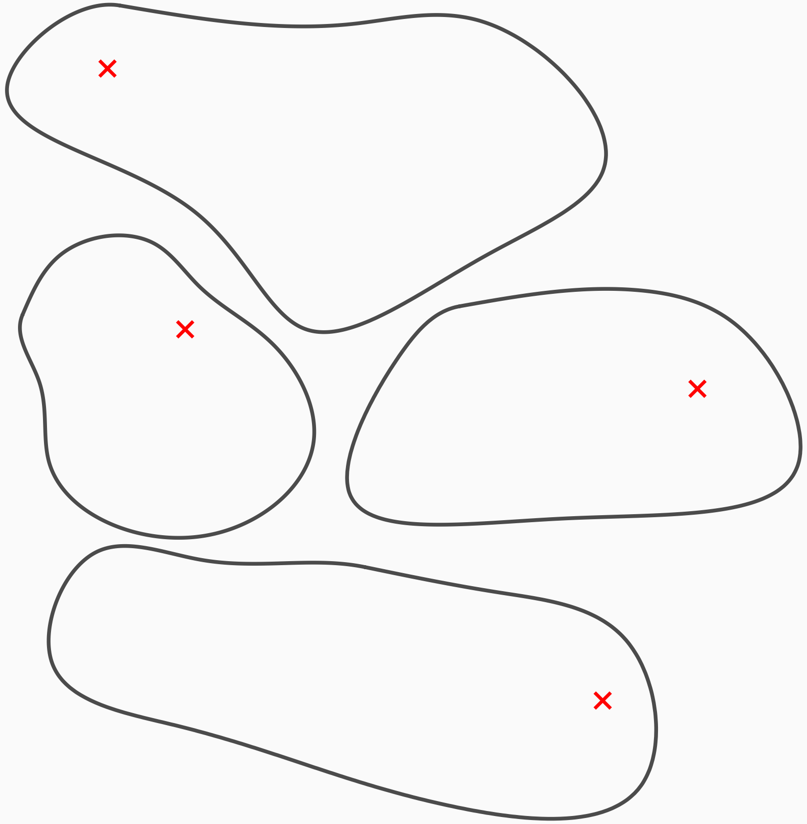


Discrepancy and approximations

ε-approximations: $A \subseteq X$ s.t. $\forall F \in \mathcal{F}, \left|\frac{|F|}{n} - \frac{|F \cap A|}{|A|}\right| \leq \varepsilon$



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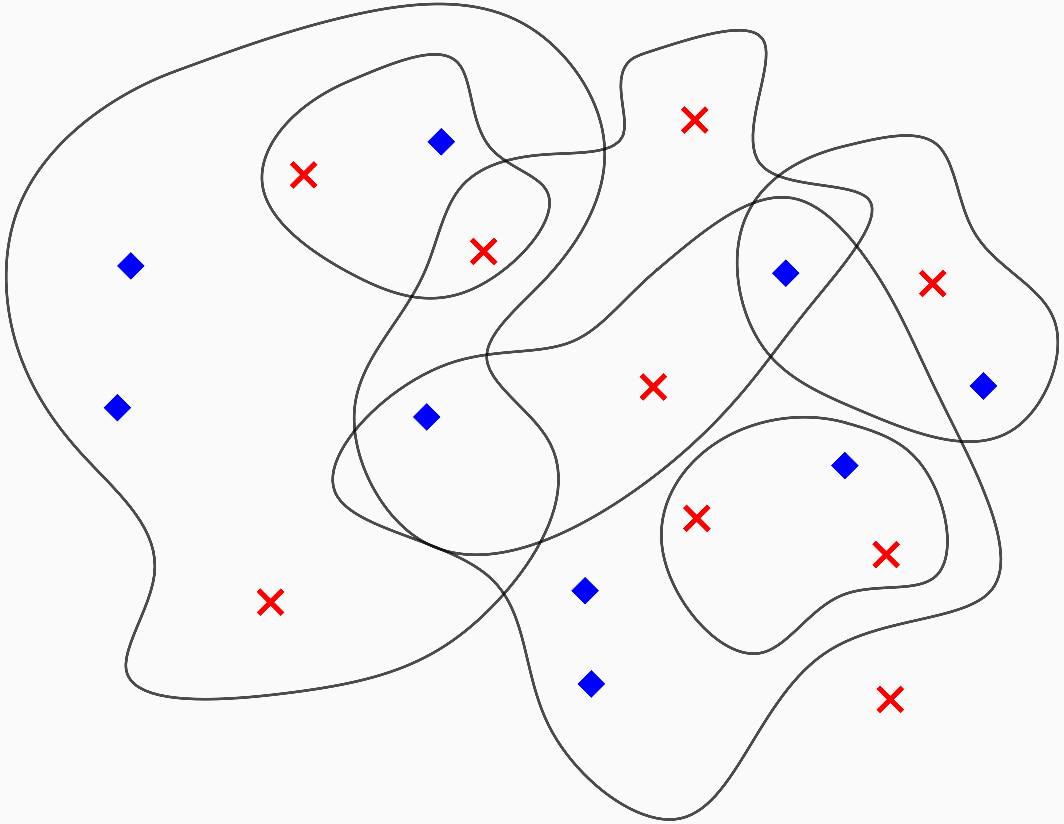

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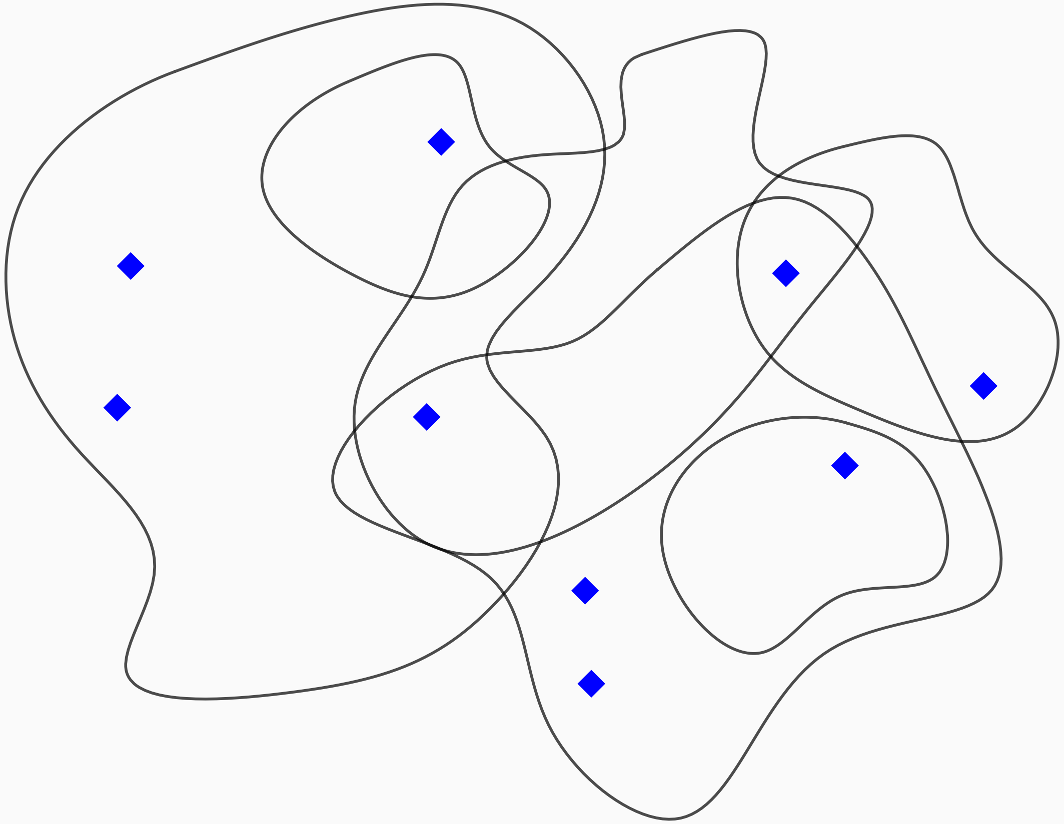

Some existing results on discrepancy

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Deterministic algorithm matching Spencer's bound in polynomial time **Levy, Ramadas, Rothvoss (2017)**

Online Discrepancy Game

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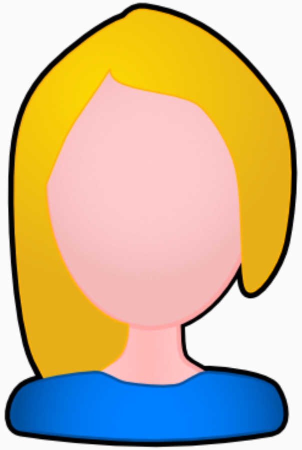
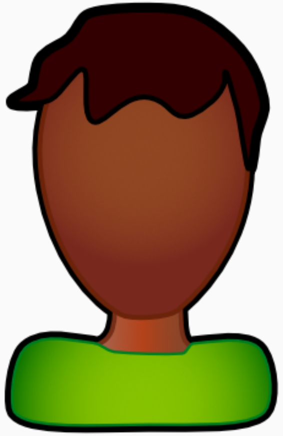


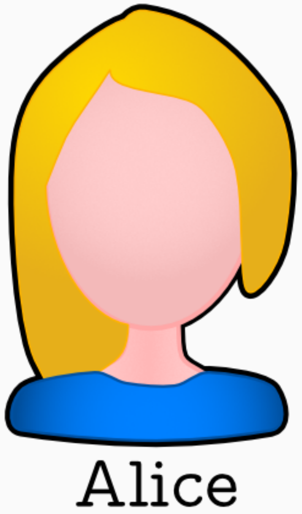
Alice

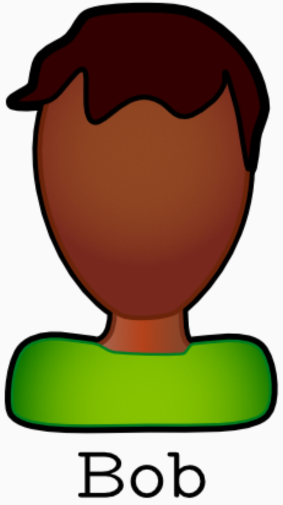


Bob

Online Discrepancy Game

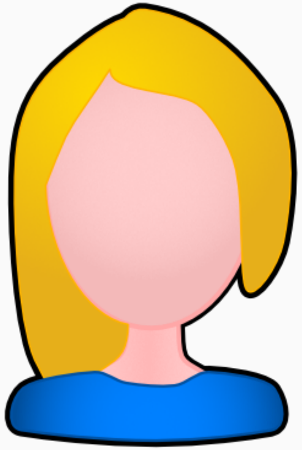
X





X

$X, \mathcal{F}$



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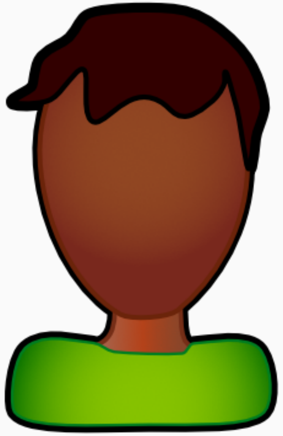
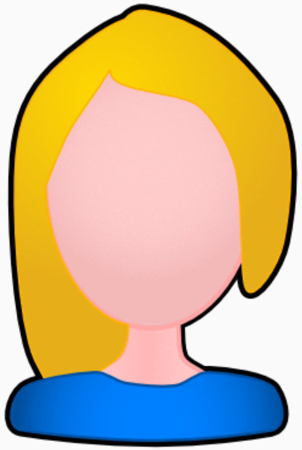


Bob

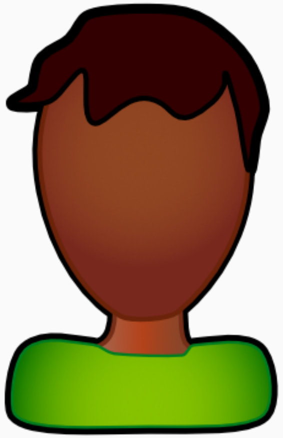
Online Discrepancy Game

X $\qquad\qquad\qquad\qquad$ $X, \mathcal{F}$



Alice

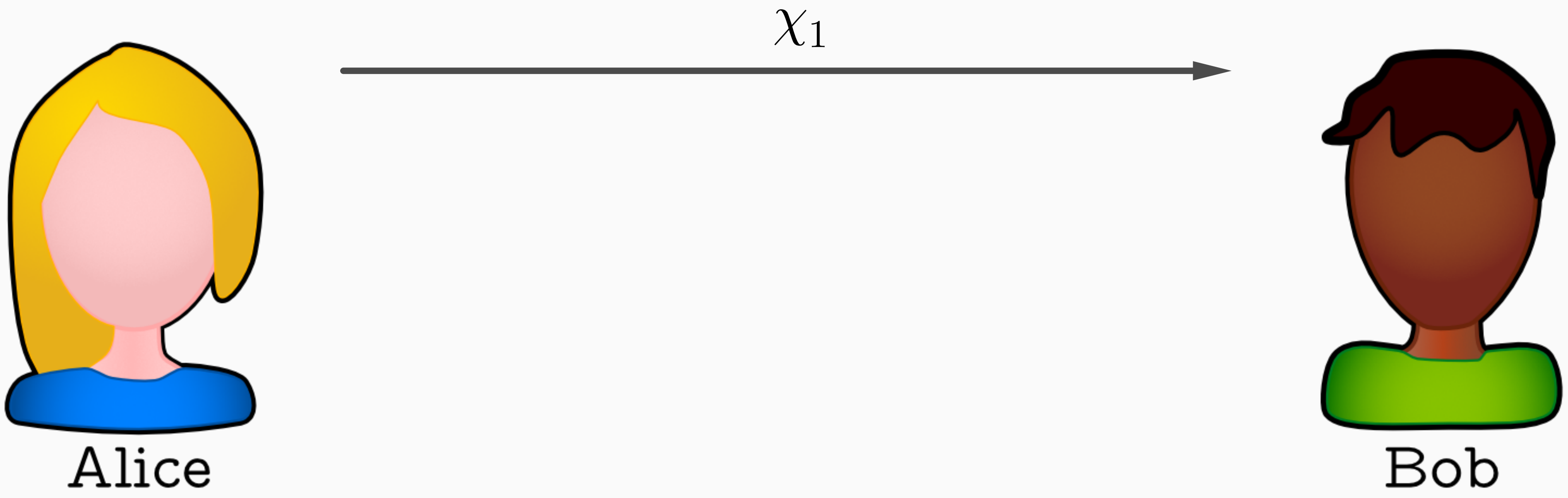
$\xrightarrow{x_1}$



Bob

X

$X, \mathcal{F}, \chi_1$



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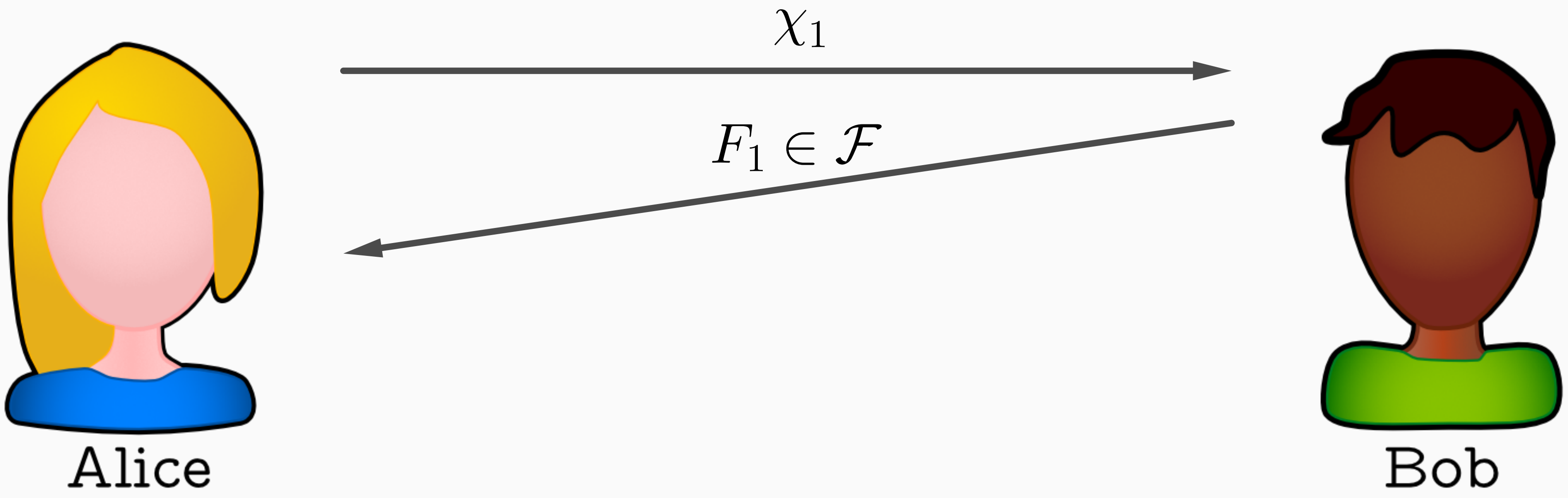

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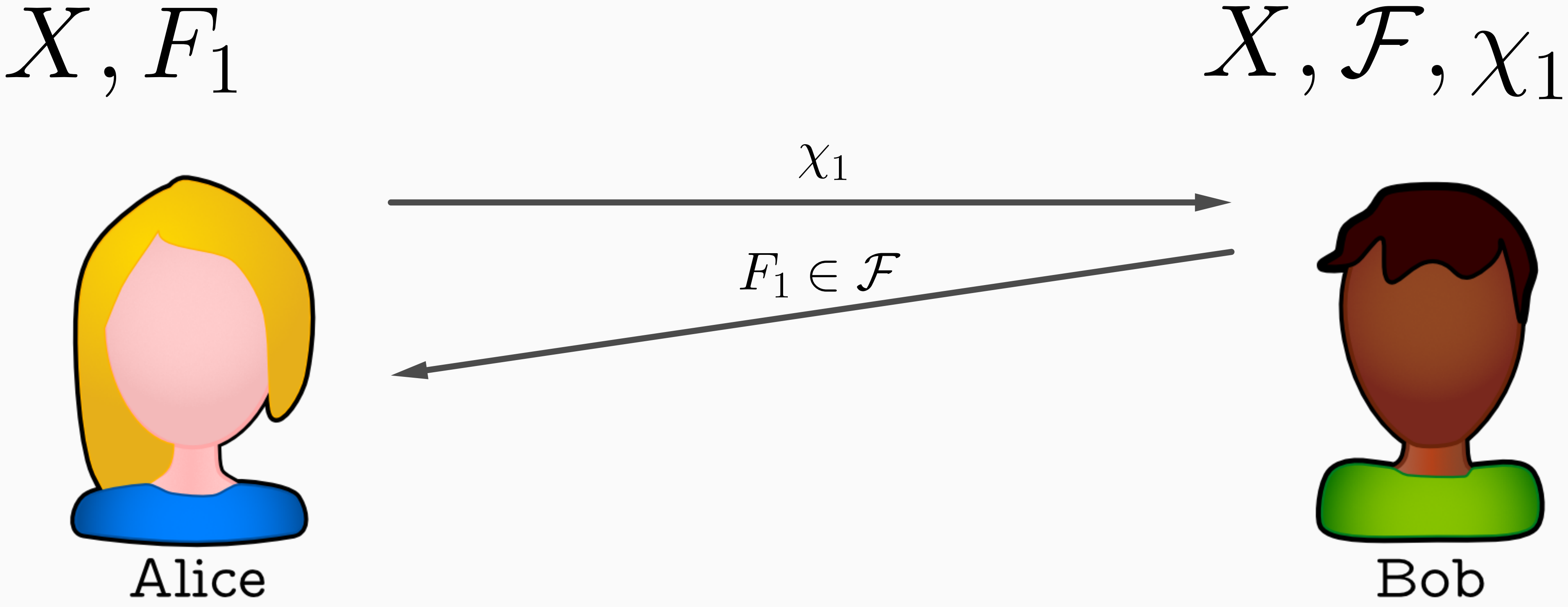
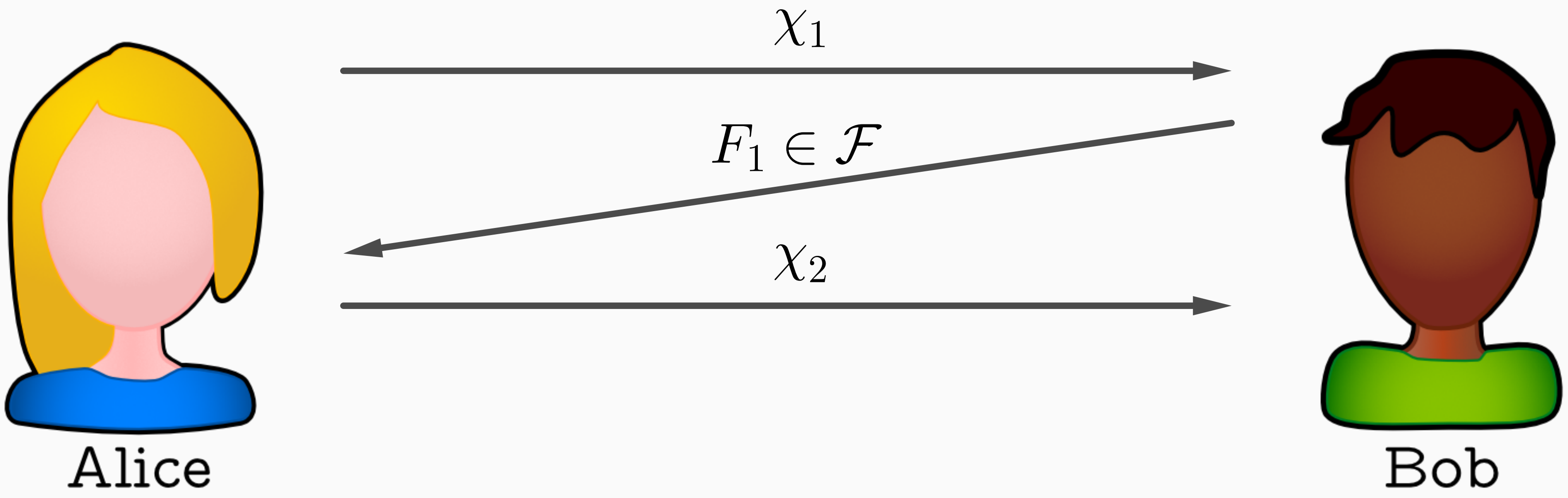


Online Discrepancy Game

X, F_1 $\qquad$ $X, \mathcal{F}, \chi_1$



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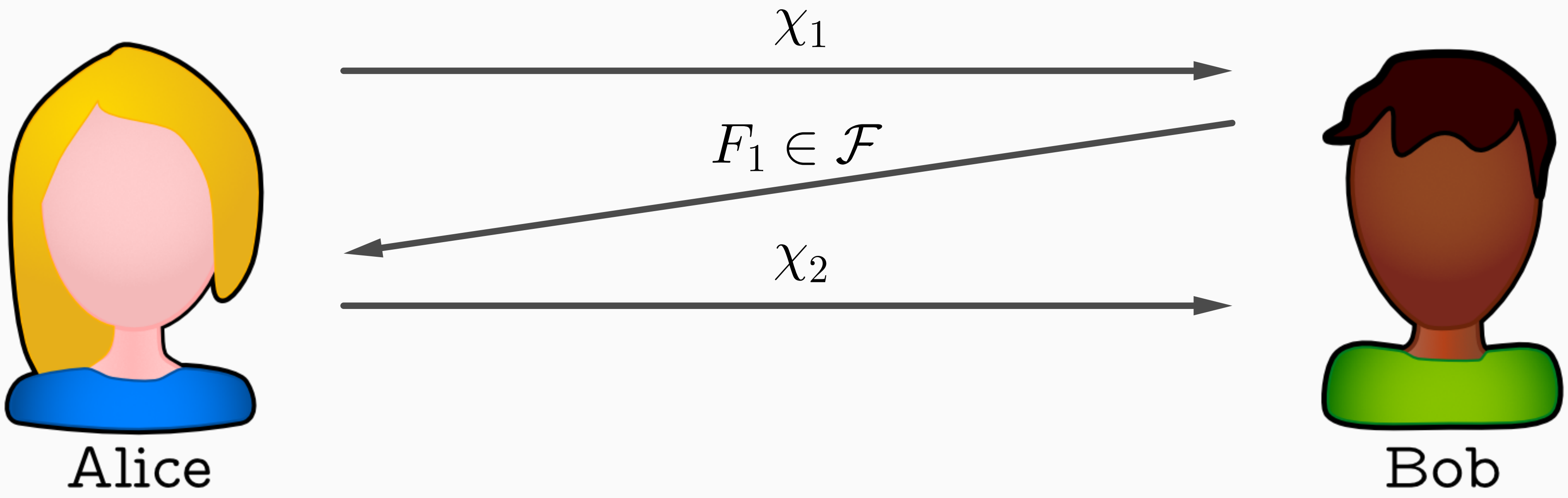

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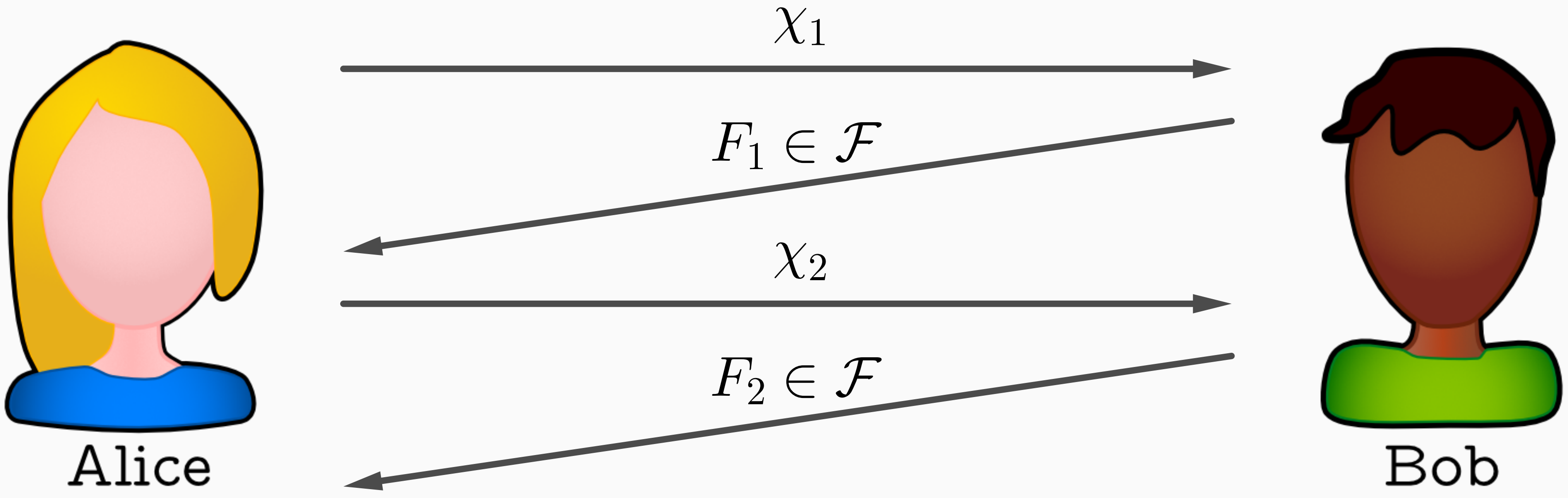

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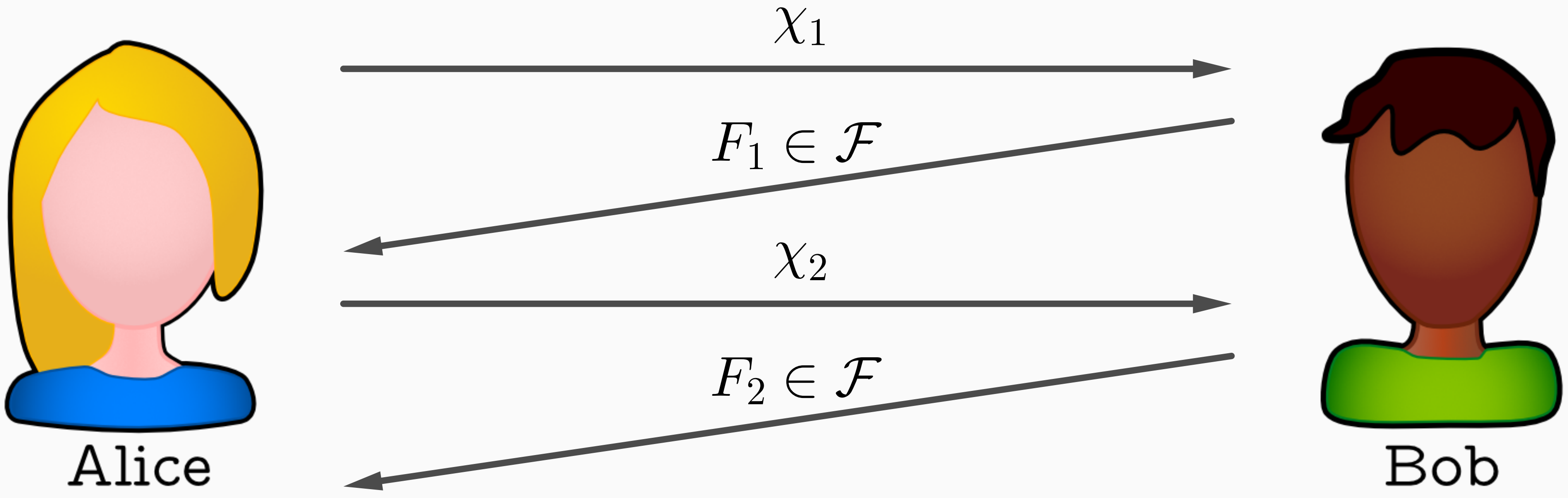

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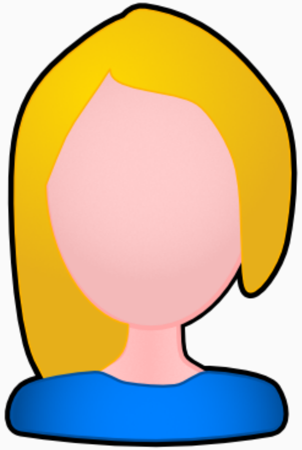

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χ_1

$F_1 \in \mathcal{F}$

χ_2

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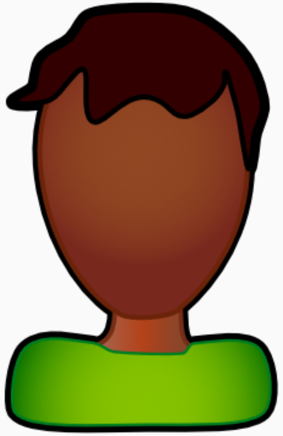

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Goal of Alice: minimize $\displaystyle \sup_{(F_1,\dots,F_T)\in\mathcal{F}} \frac{1}{T}\sum_{t=1}^{T} |\chi_t(F_t)|$

Interest of the online discrepancy game

Goal of Alice: minimize $\sup_{(F_1,\ldots,F_T)\in\mathcal{F}} \frac{1}{T}\sum_{t=1}^{T}|\chi_t(F_t)| \geq$ **offline coloring bound**

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$\hookrightarrow$ Random coloring also $\tilde{\Theta}(\sqrt{n})$ for finite VC-dimension set systems

Results on ODG

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Main result. For T rounds on VC-dimension d set system

$$\frac{1}{T}\sum_{t=1}^{T}|\chi_t(F_t)| = \tilde{O}\left(\max\left(\, T^{-\frac{1}{2d}}\sqrt{n}\ ,\ n^{\frac{1}{2}-\frac{1}{2d}}\,\right)\right)$$

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Remark. We can compute $\chi_1, \ldots, \chi_{\frac{n}{16}}$ s.t.

$$\forall F \in \mathcal{F}, \frac{16}{n}\mathbb{E}\left[\sum_{t=1}^{\frac{n}{16}}|\chi_t(F)|\right] = \tilde{O}\left(n^{\frac{1}{2}-\frac{1}{2d}}\right)$$

where χ_t is computed knowing only $F_1, \ldots, F_{t-1}$.

Key tools: Multiplicative Weight Update, ε-approximations, Freedman's inequality

ODG: Alice's strategy

Theorem (Lovett, Meka (2015)) Given $\mathcal{F}$ with $m = O(n)$, the algorithm computes a random coloring $\mathrm{LM}_{\mathcal{F}}$ s.t.

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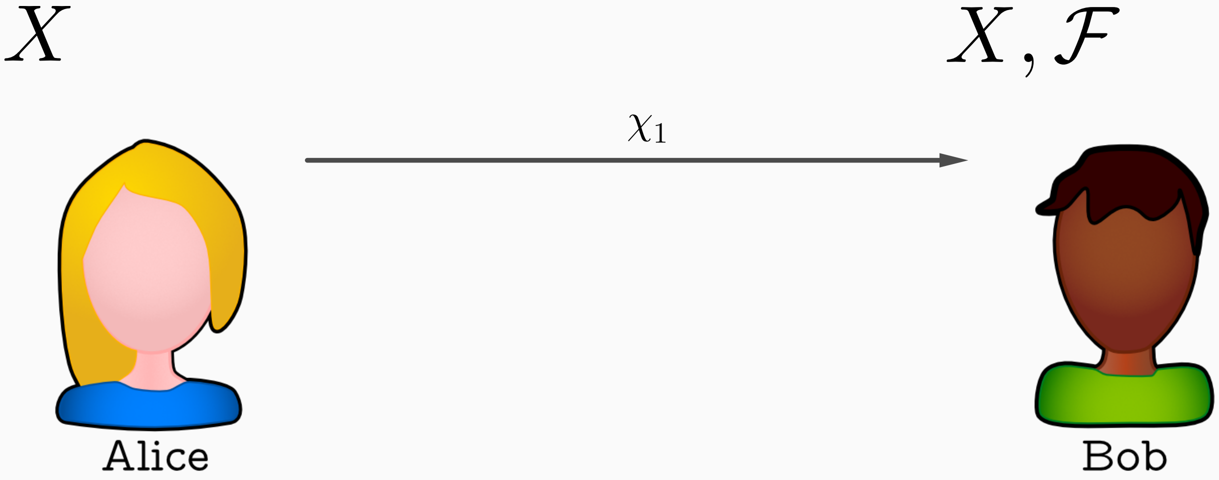

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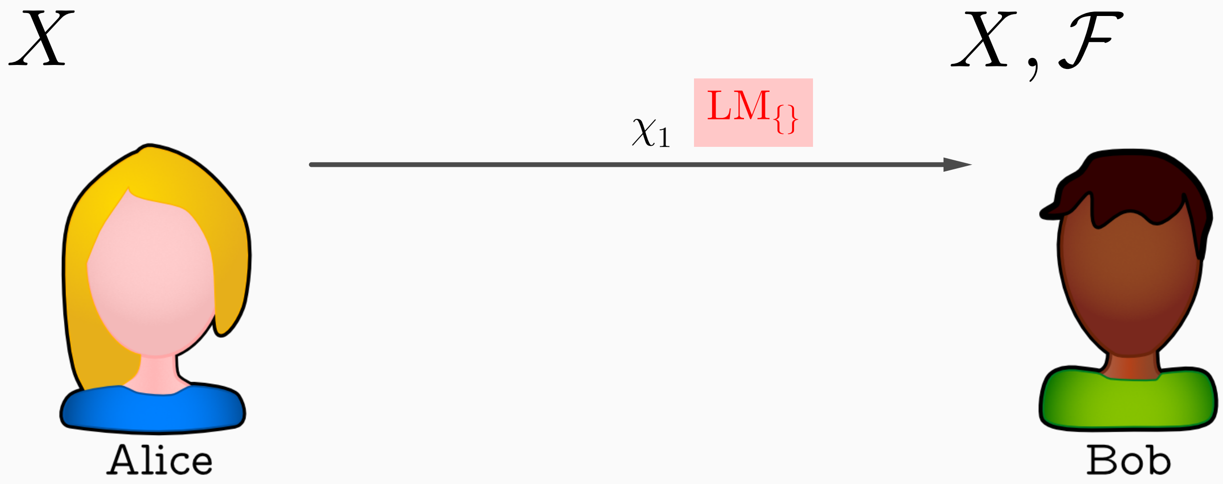

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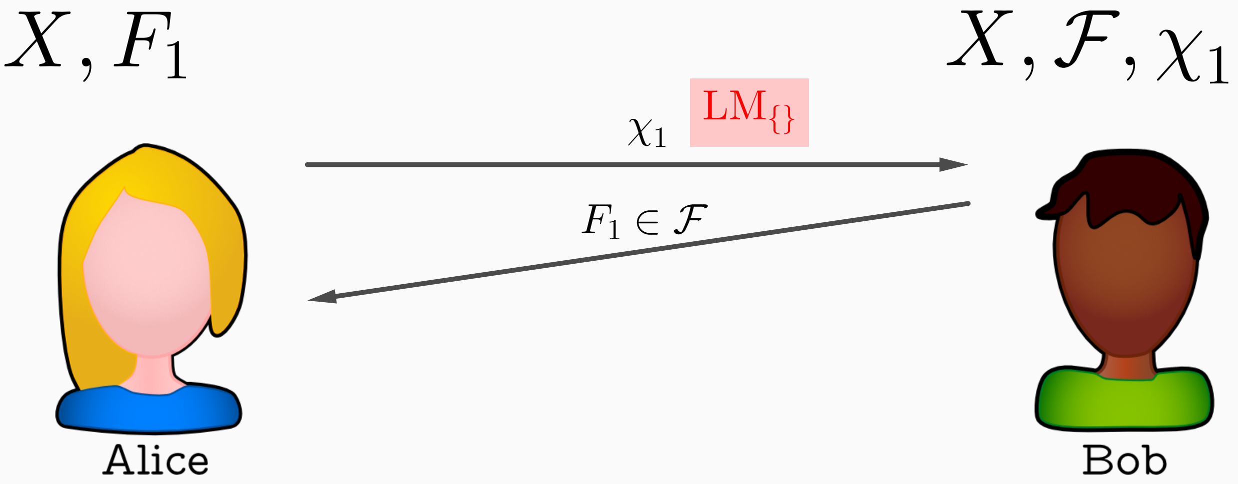

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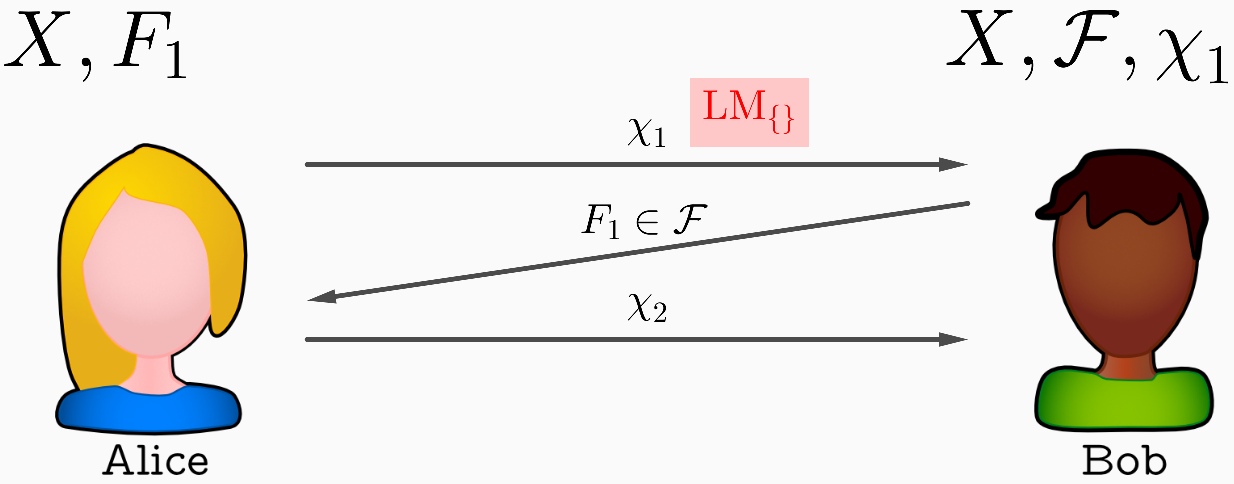

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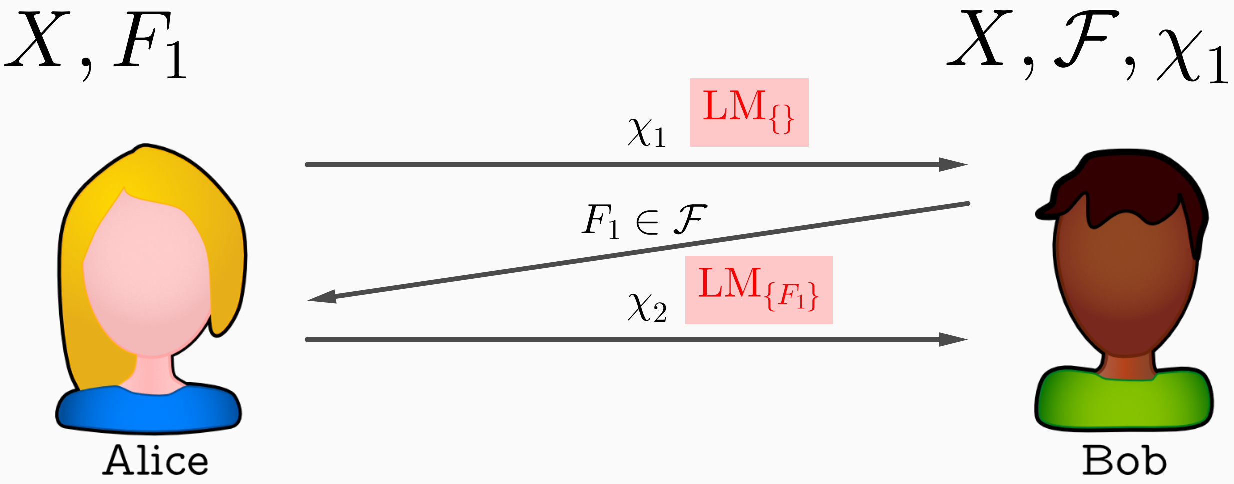

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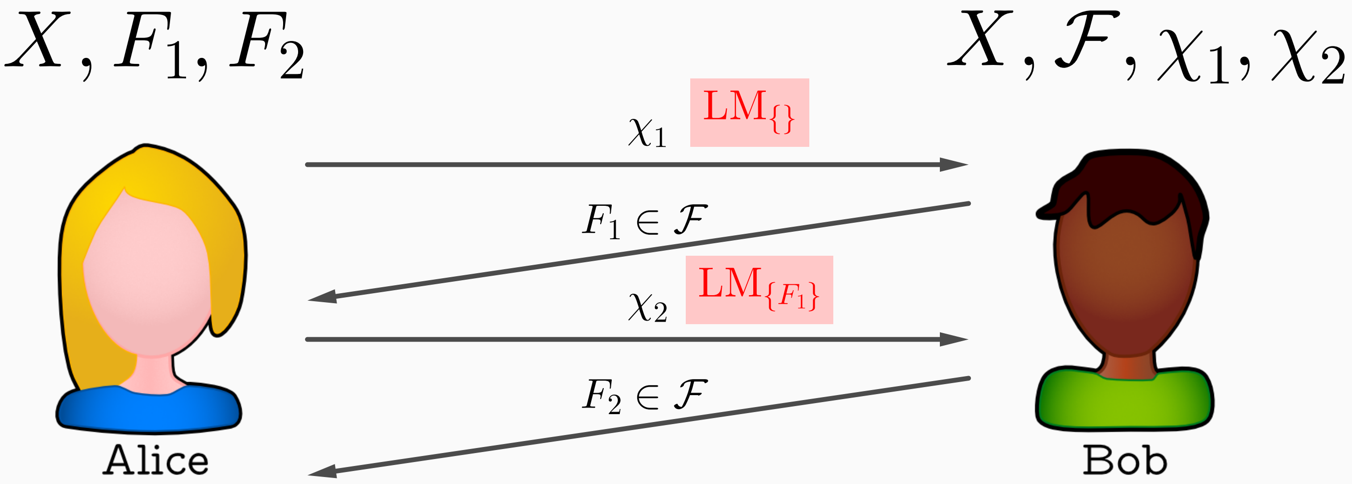

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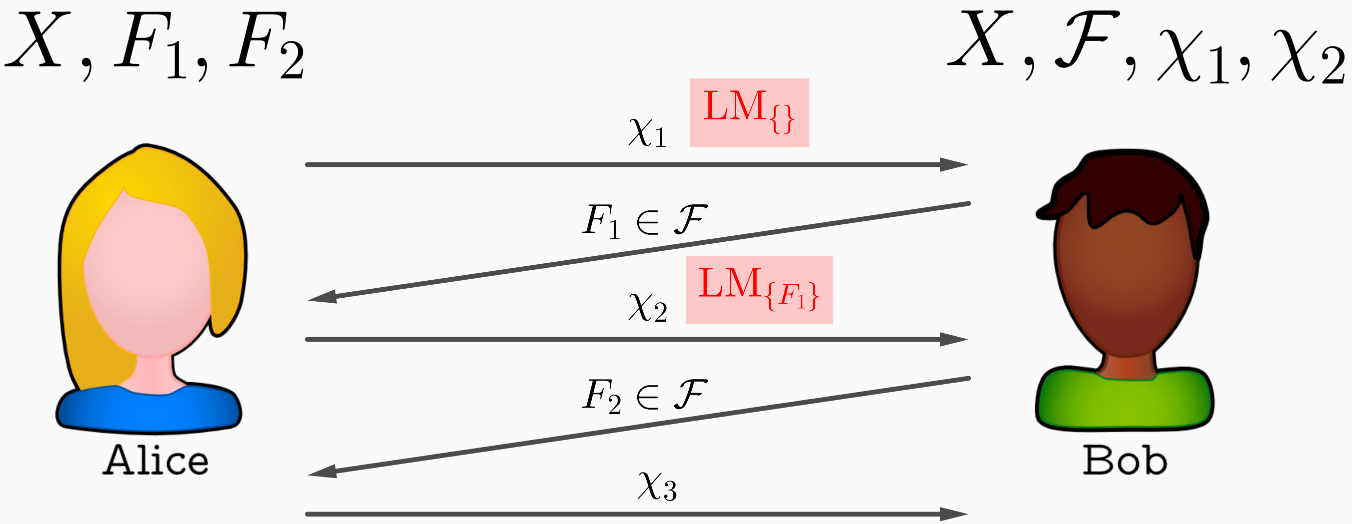

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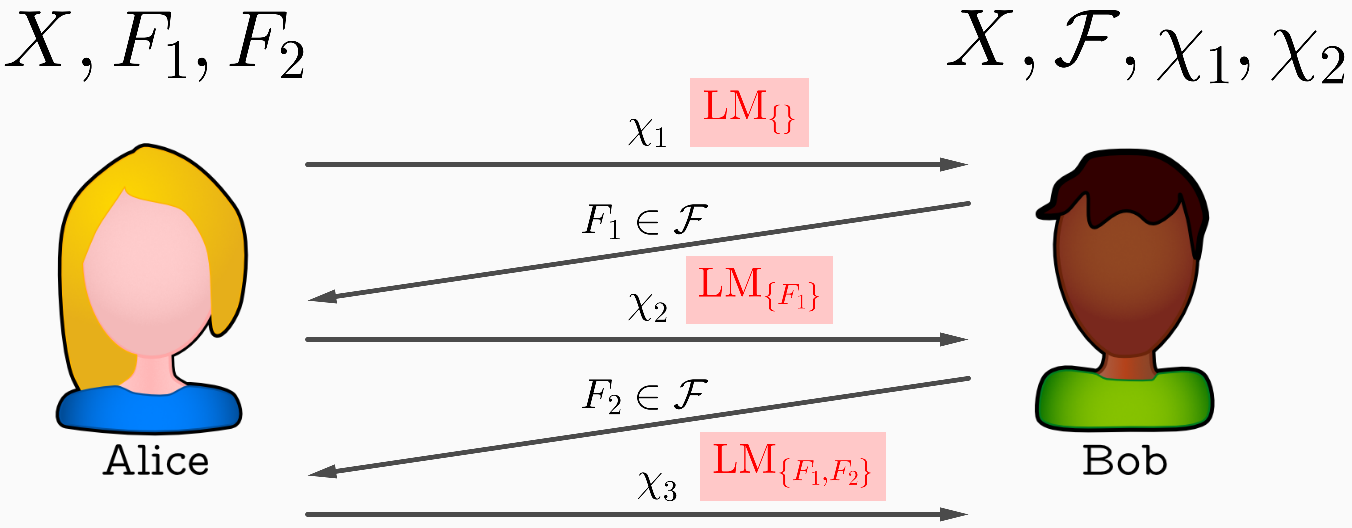

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At iteration $t \leq n$, Alice choses $\chi_t = \mathrm{LM}_{\{F_1,\ldots,F_{t-1}\}}$

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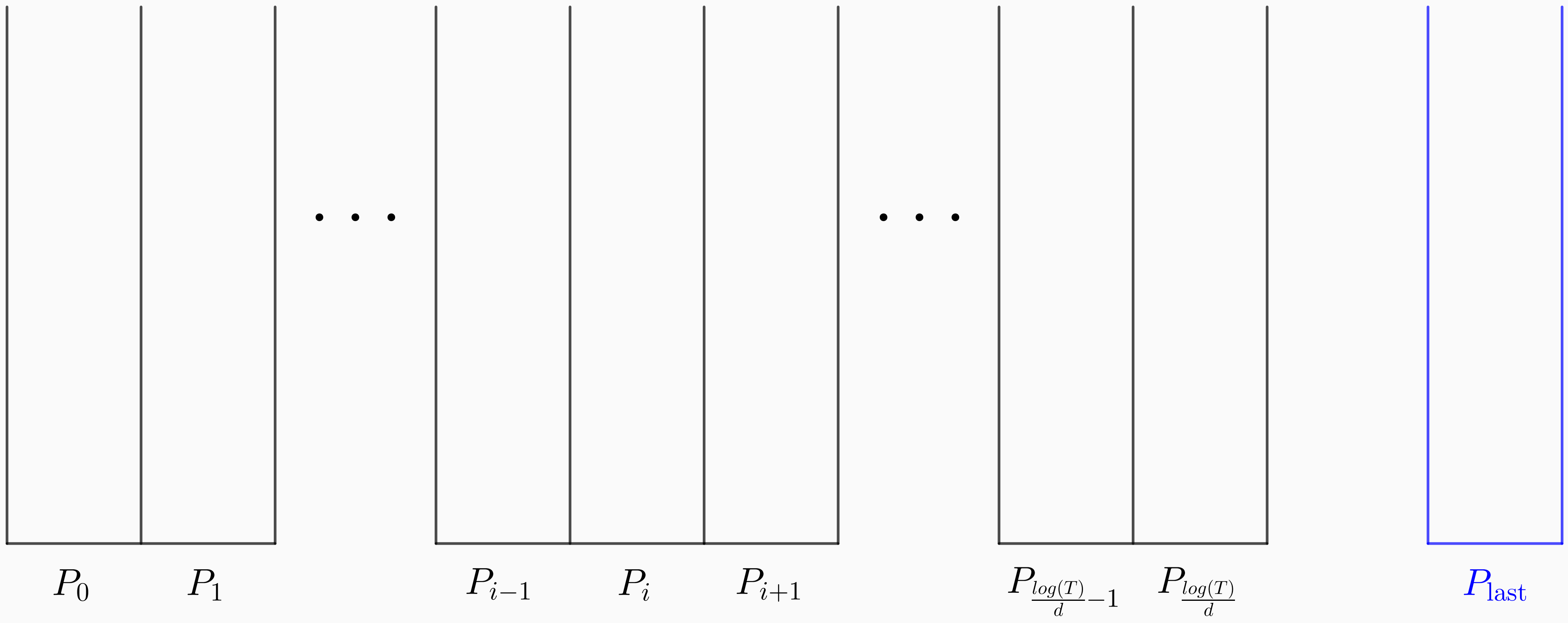
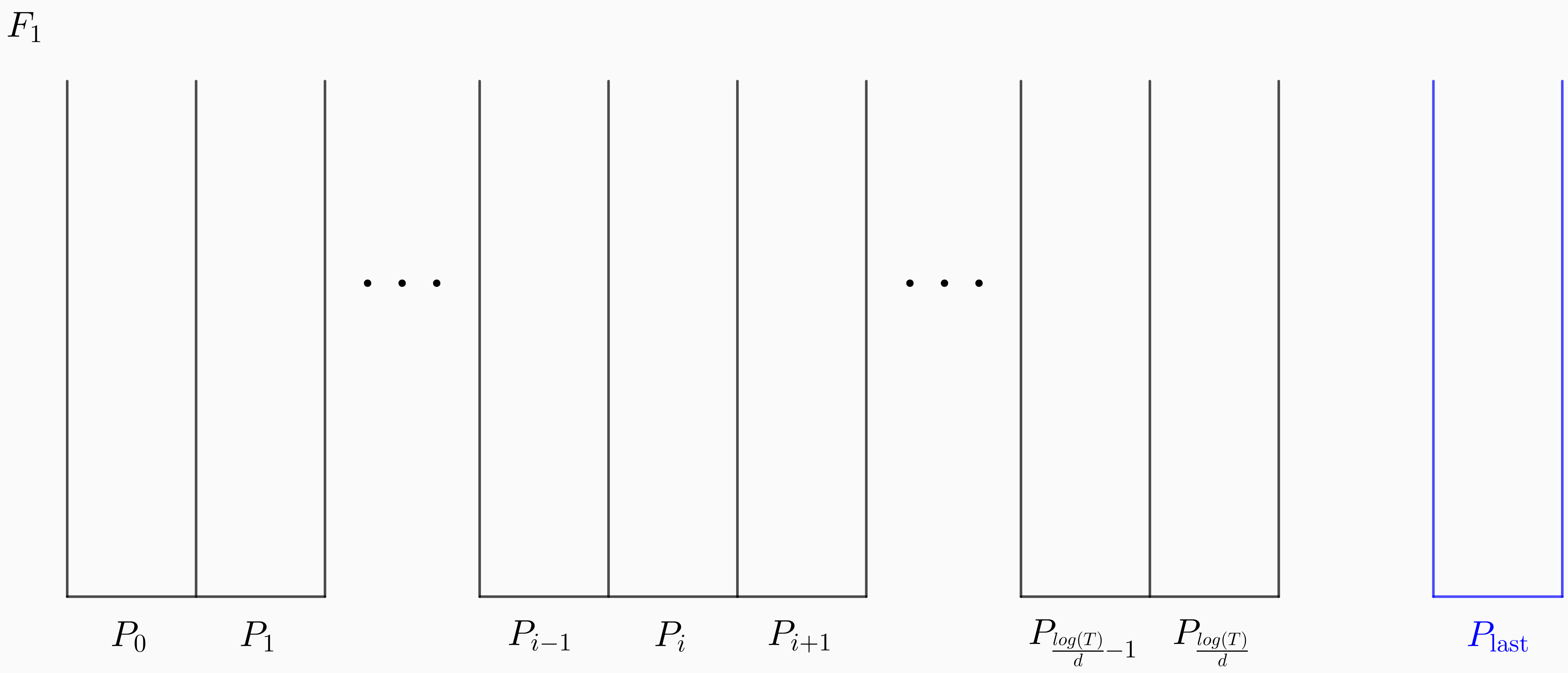
At iteration $t \leq n$, Alice choses $\chi_t = \mathrm{LM}_{\{F_1,\ldots,F_{t-1}\}}$

$$\Rightarrow \chi_t(F_1) = \ldots = \chi_t(F_{t-1}) = 0$$

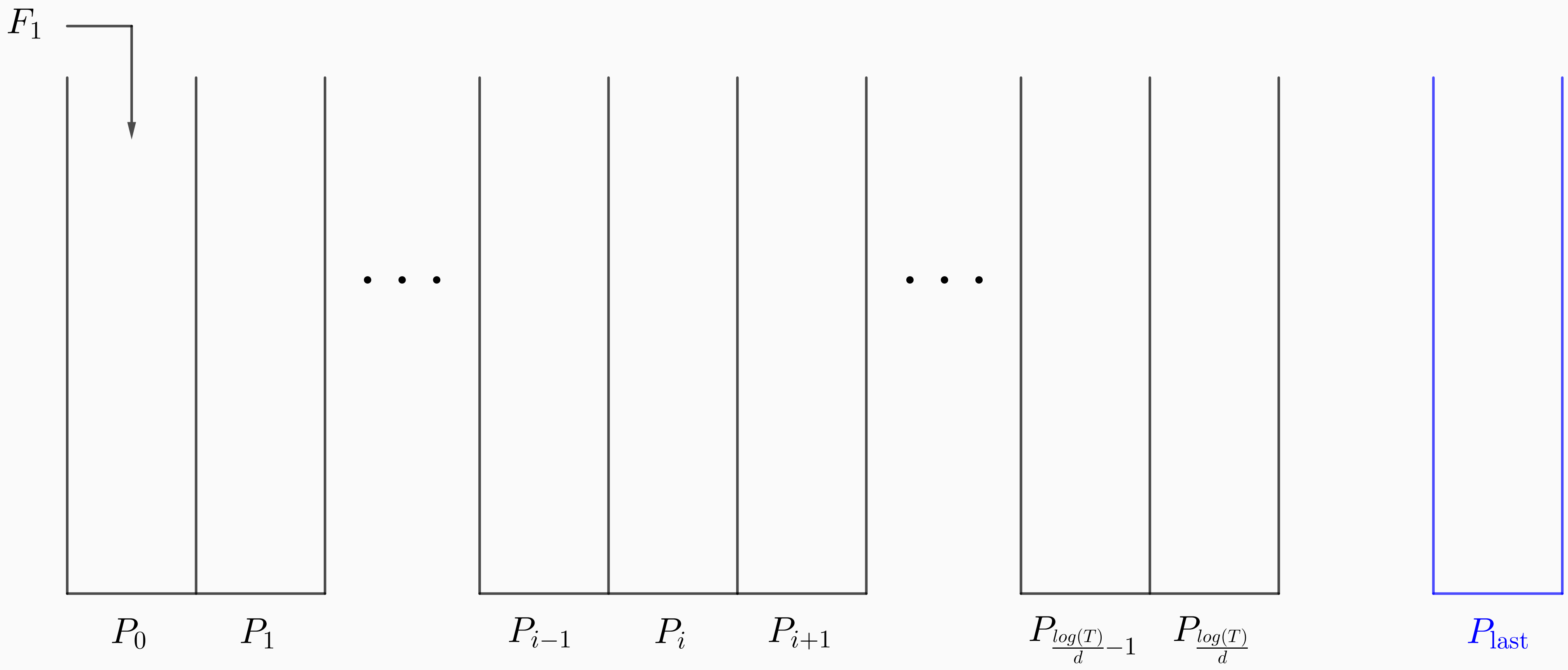
Iteration 0 :


P_0
P_1
$\dots$
P_{i-1}
P_i
P_{i+1}
$\dots$
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration 1 :


F_1
$\dots$
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration 1 :



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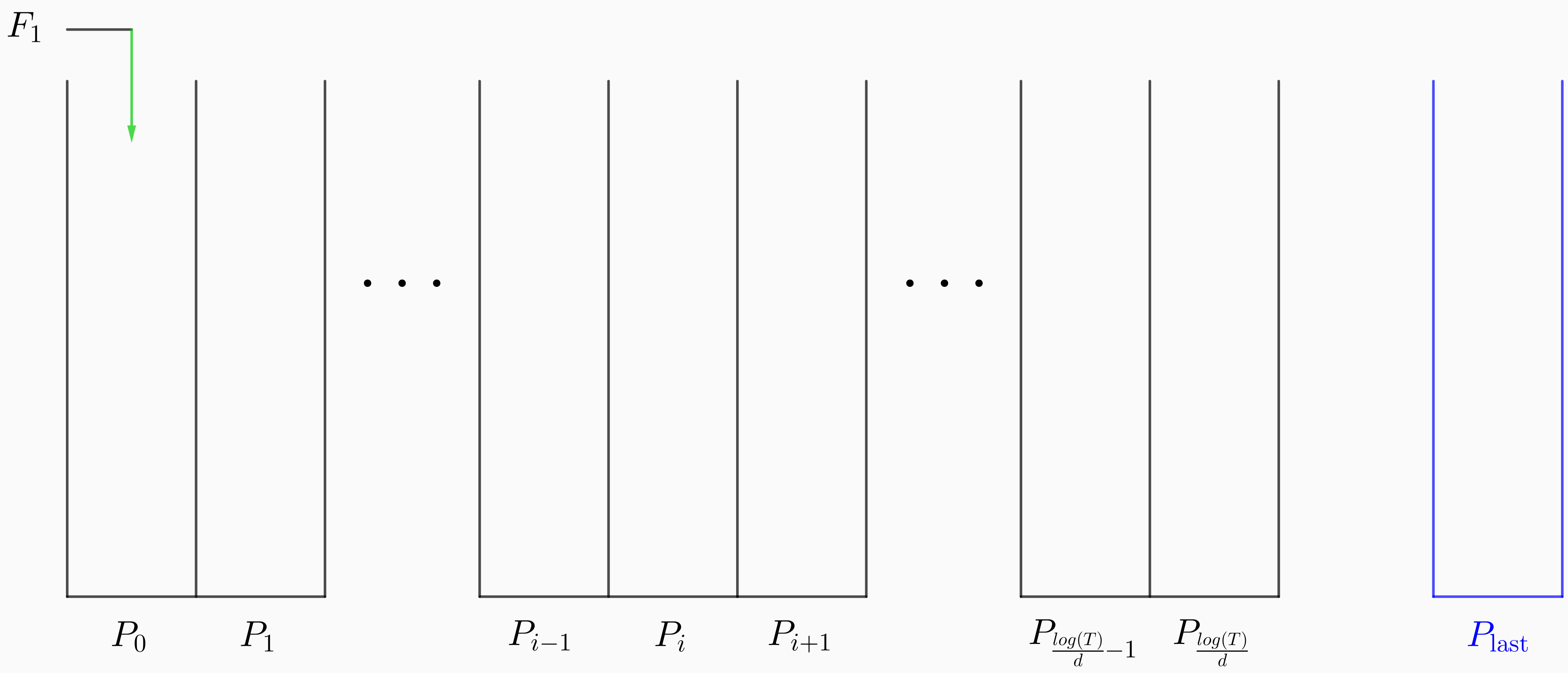
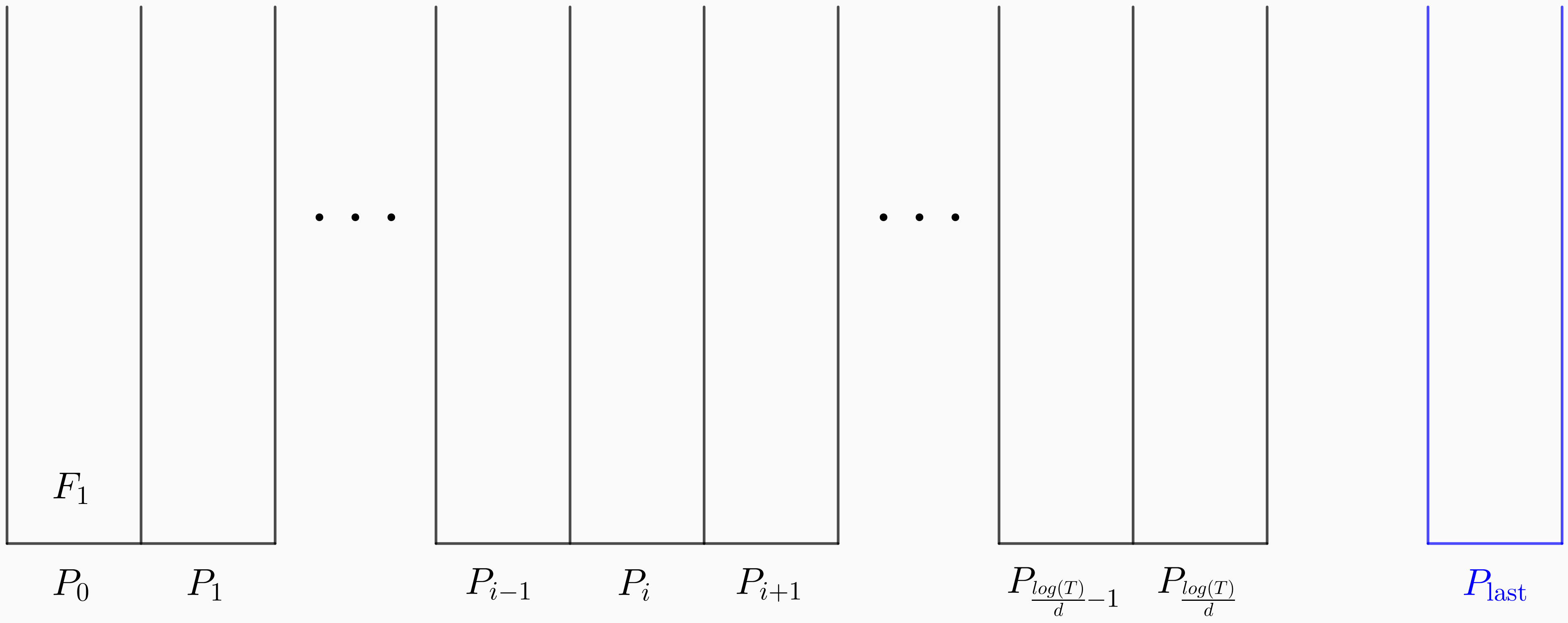
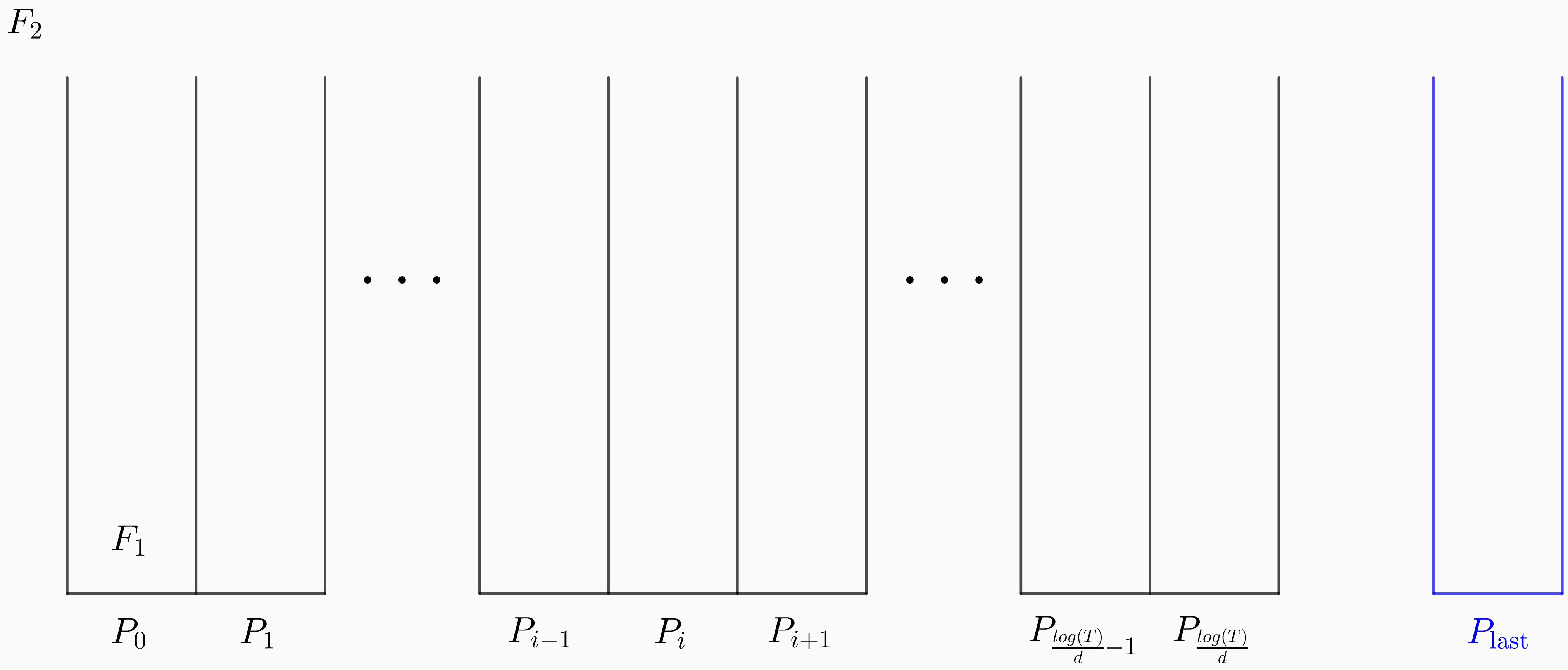


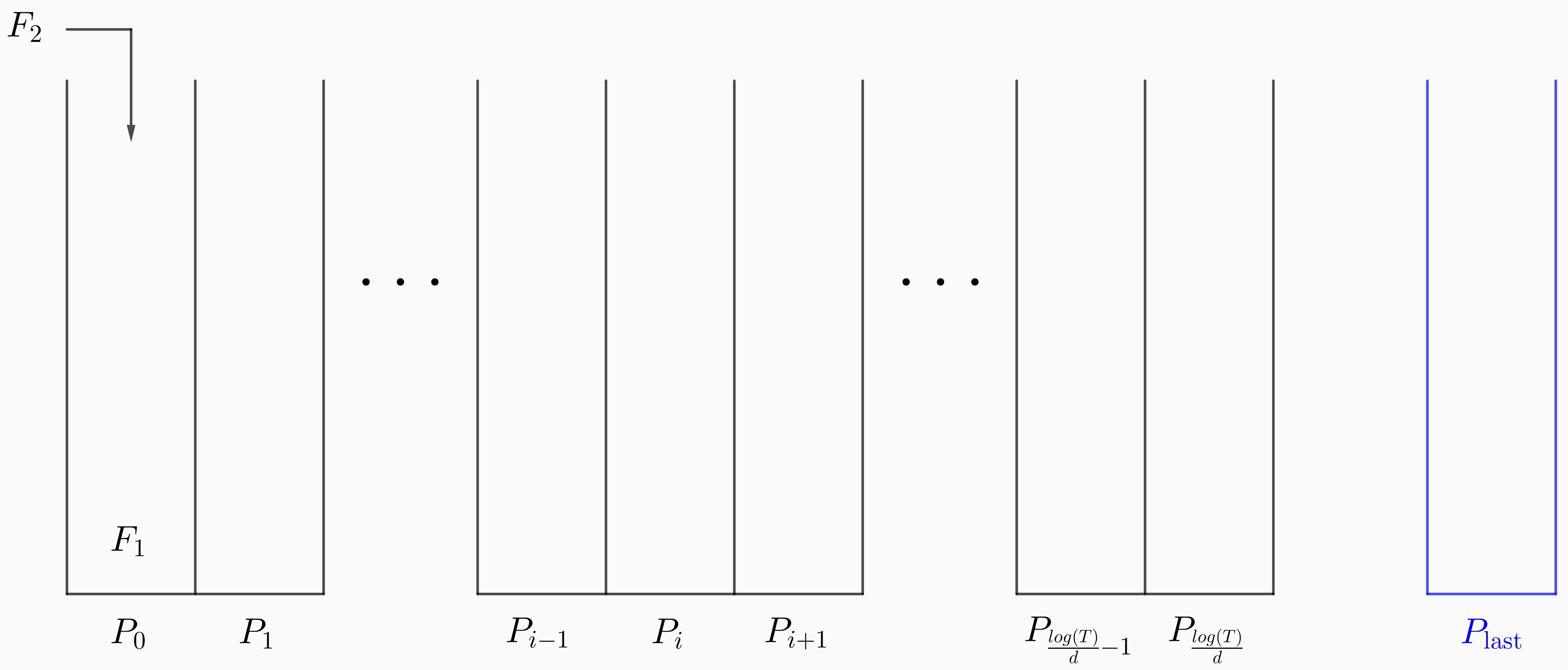
Iteration 1 :


F_1
P_0
P_1
$\dots$
P_{i-1}
P_i
P_{i+1}
$\dots$
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration 2 :

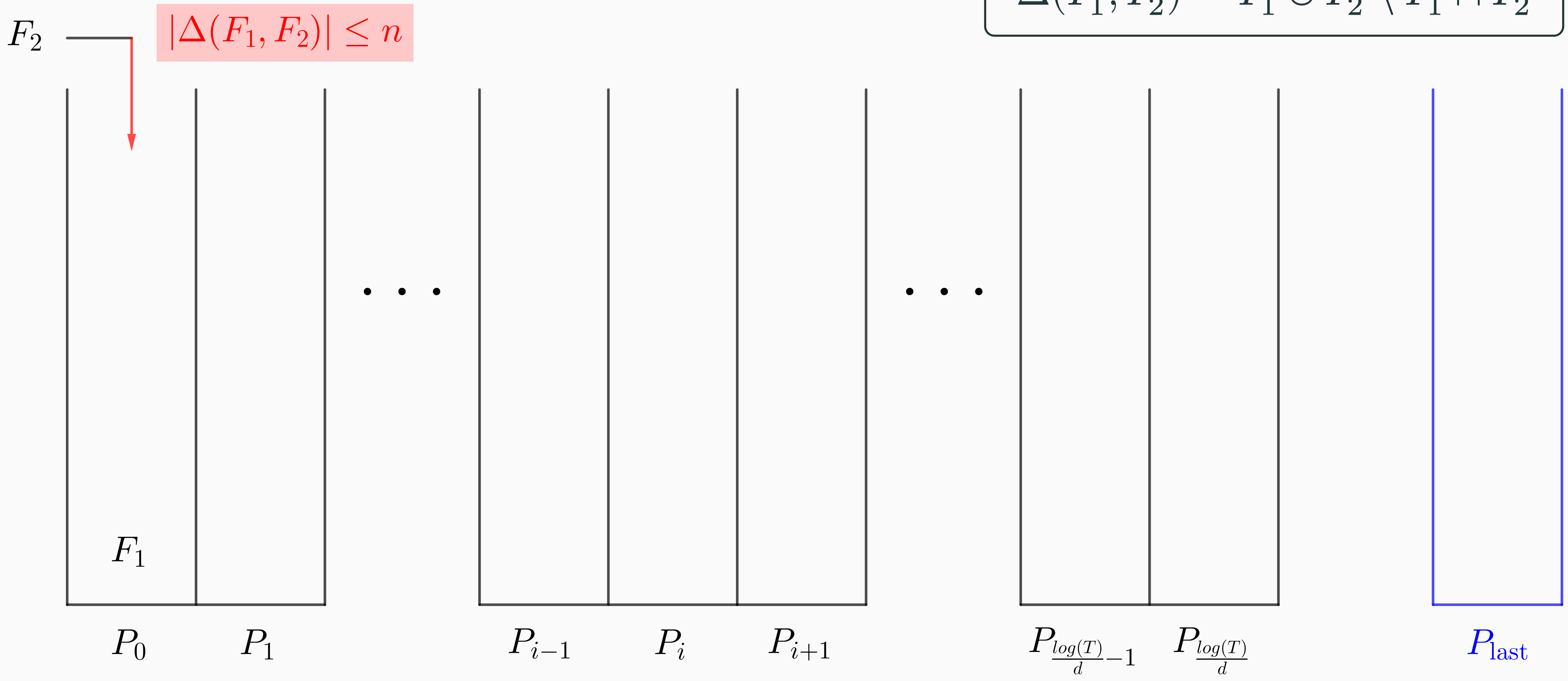

F_2
F_1
P_0
P_1
$\dots$
P_{i-1}
P_i
P_{i+1}
$\dots$
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration 2 :

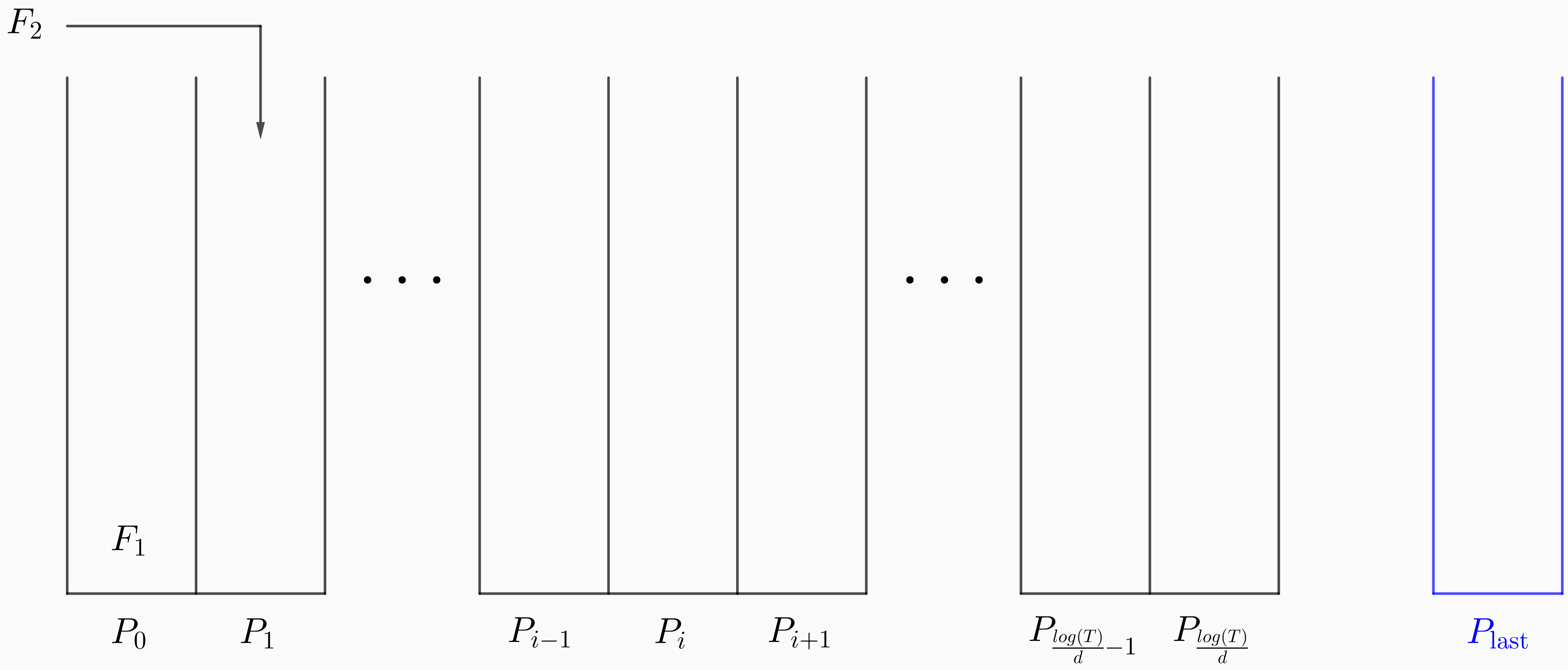

F_2
F_1
$\cdots$
$\cdots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration 2 :

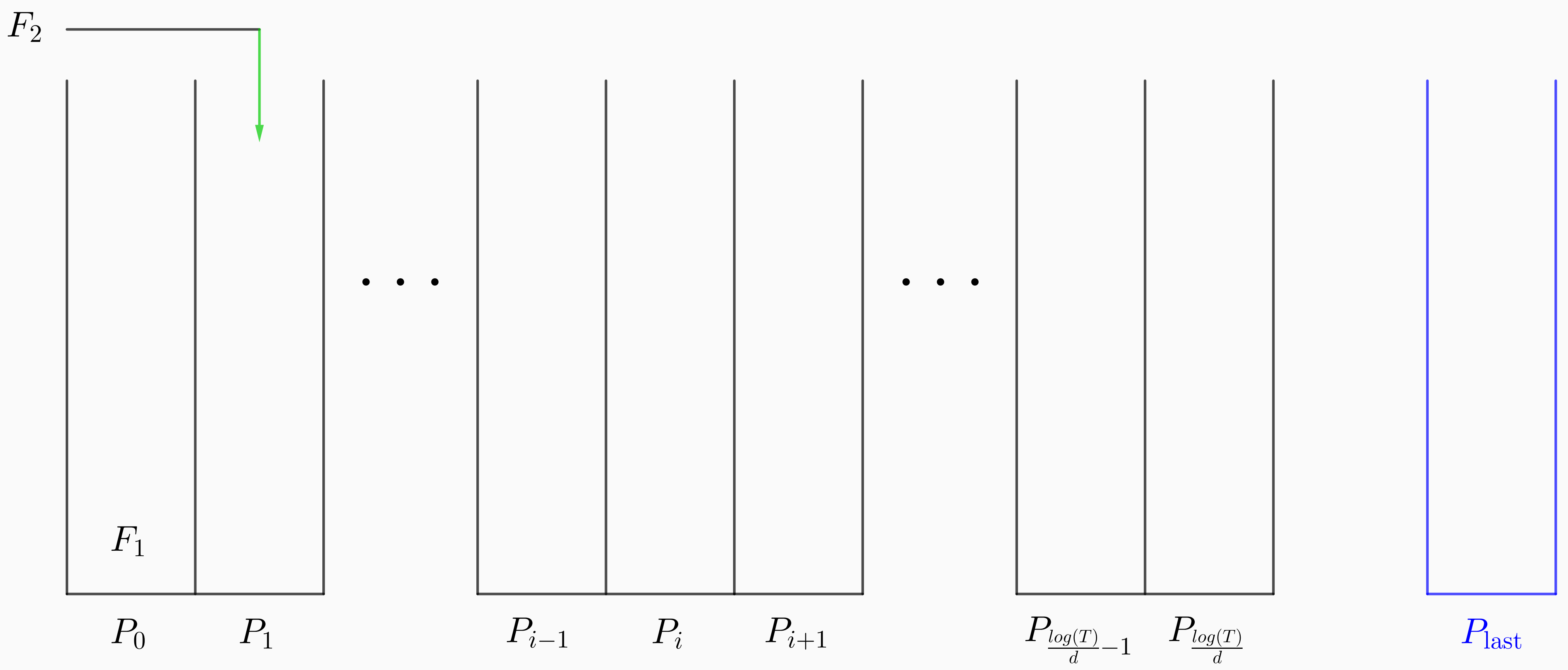
$$\boxed{\Delta(F_1, F_2) = F_1 \cup F_2 \setminus F_1 \cap F_2}$$



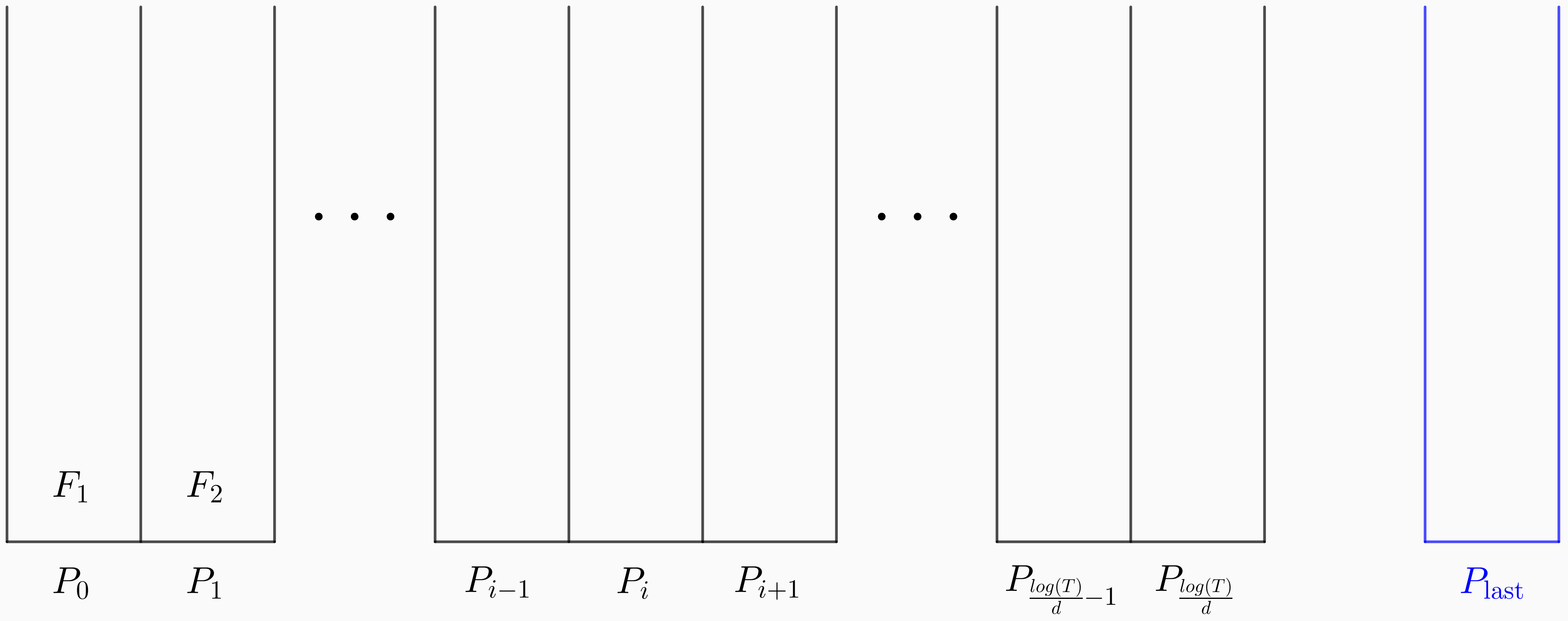
Iteration 2 :


F_2
F_1
$\dots$
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

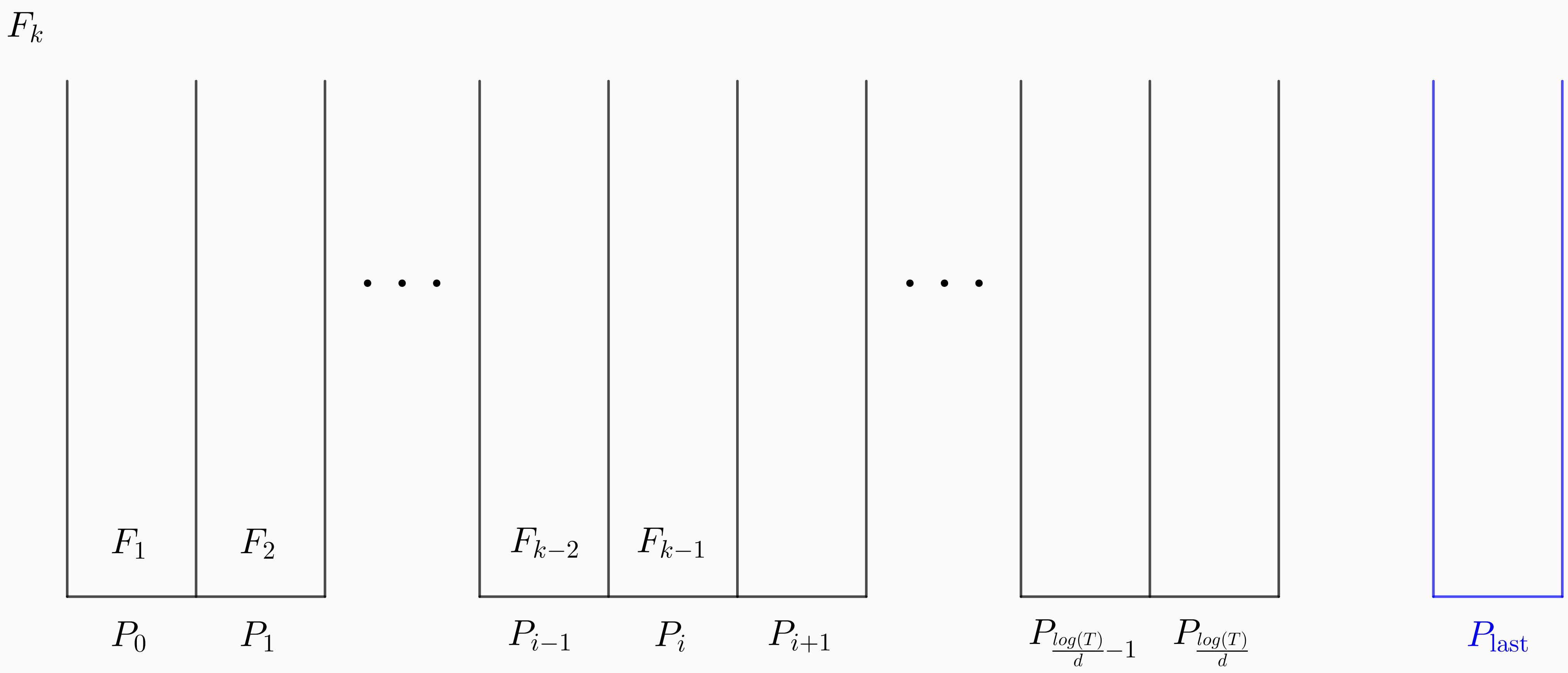
Iteration 2 :


F_2
F_1
$\dots$
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

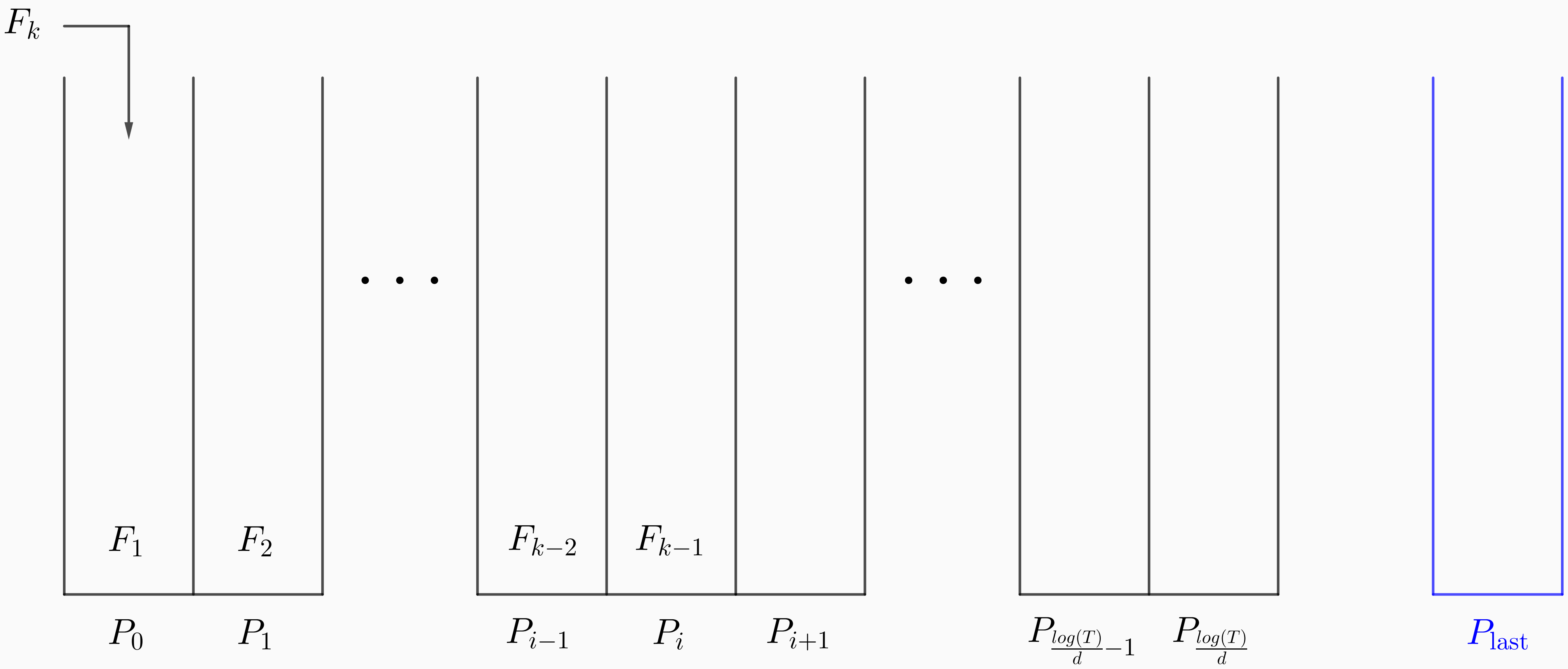
Iteration 2 :


F_1
F_2
$\dots$
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

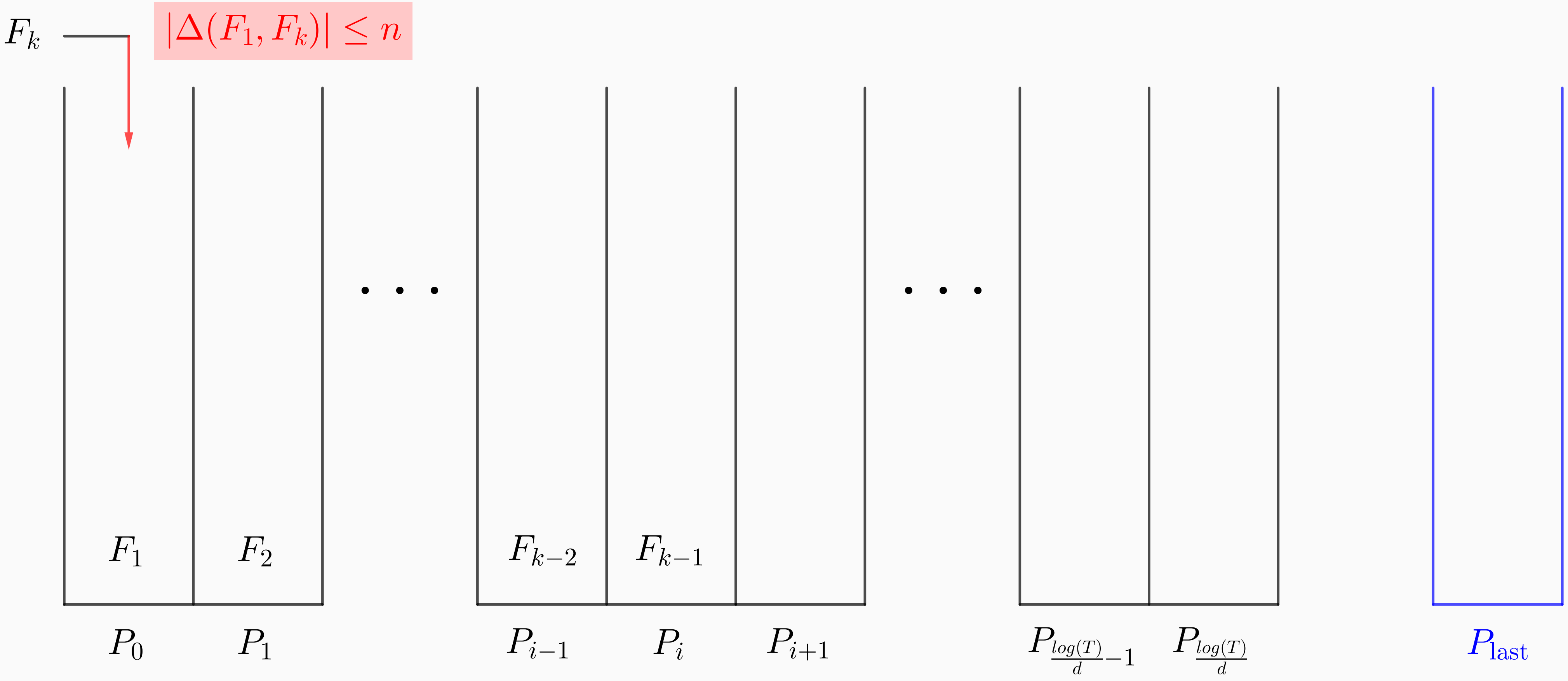
Iteration k :


F_k
F_1
F_2
F_{k-2}
F_{k-1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

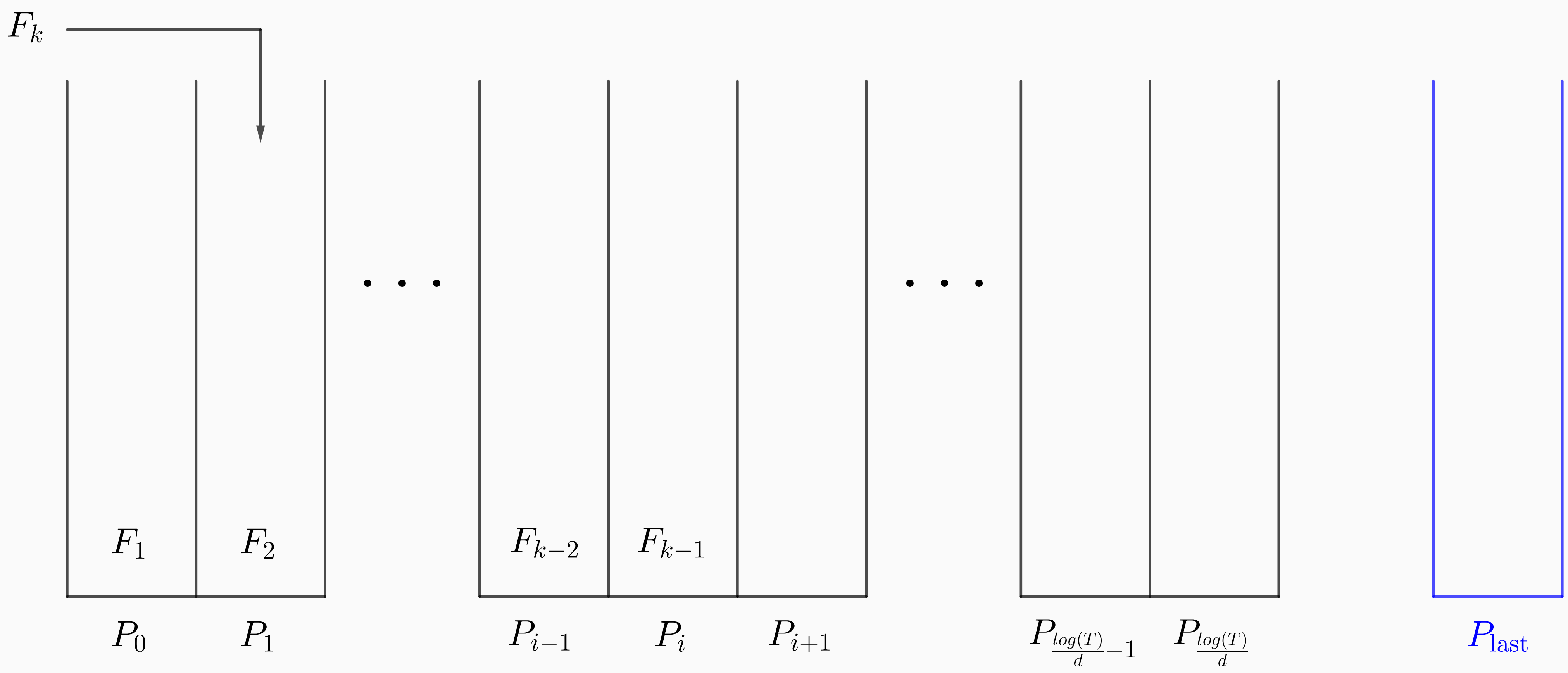
Iteration k :


F_k
F_1
F_2
F_{k-2}
F_{k-1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

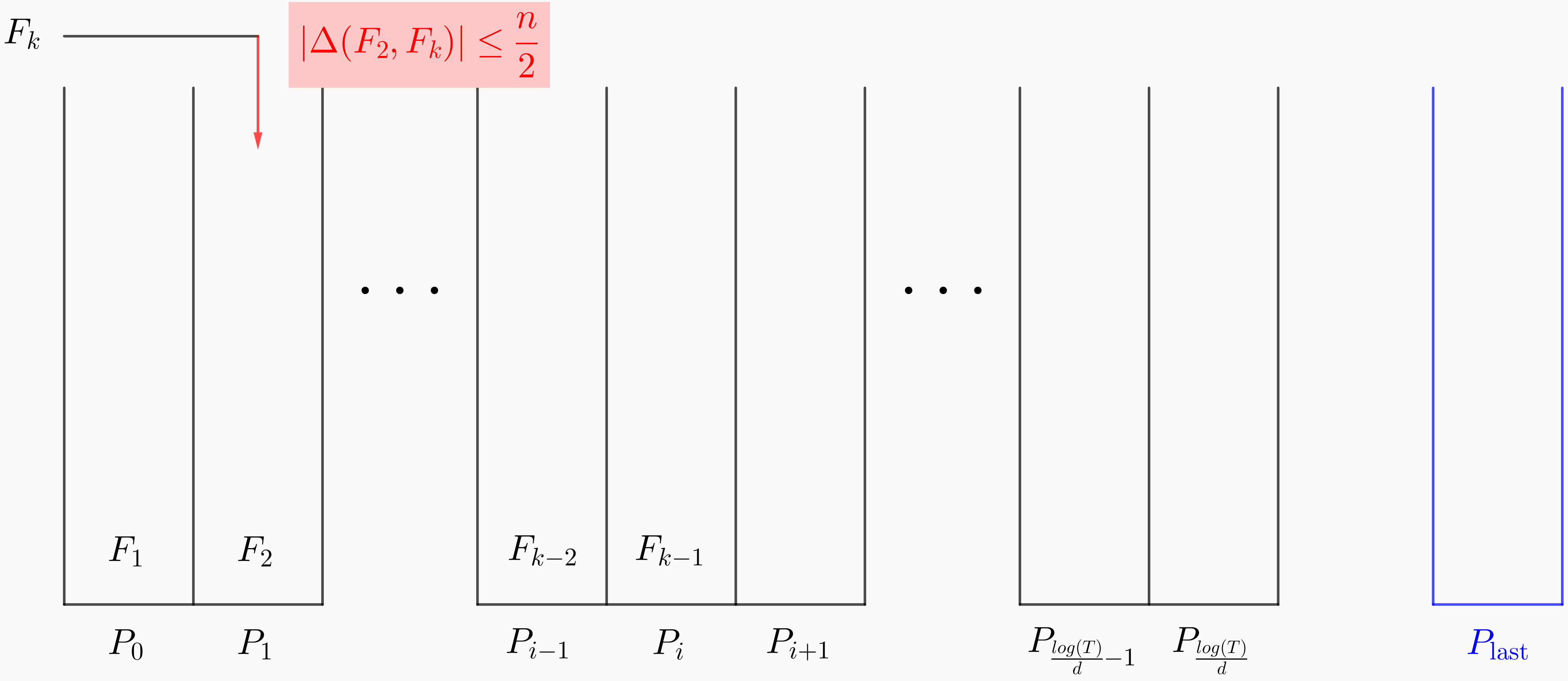
Iteration k :


F_k
$|\Delta(F_1, F_k)| \leq n$
F_1
F_2
$\dots$
F_{k-2}
F_{k-1}
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

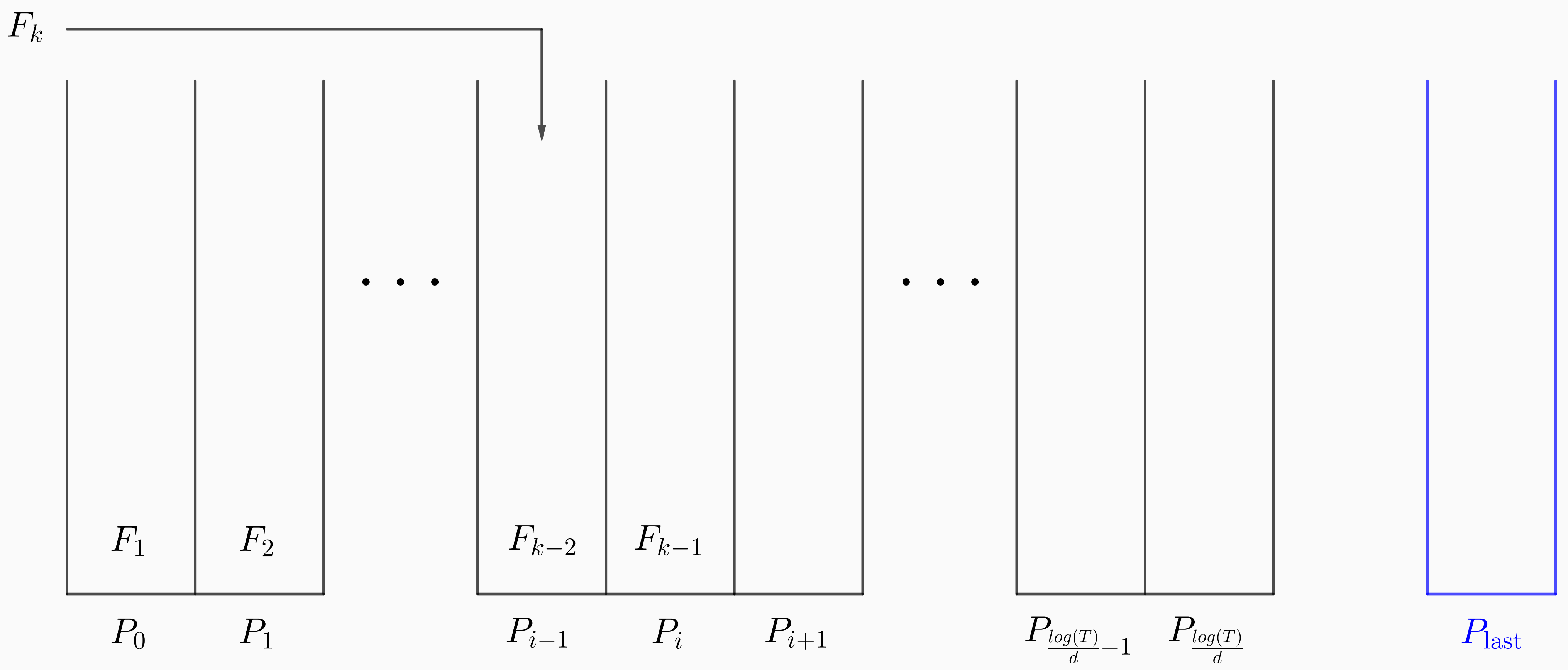
Iteration k :


F_k
F_1
F_2
F_{k-2}
F_{k-1}
$\dots$
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration k :

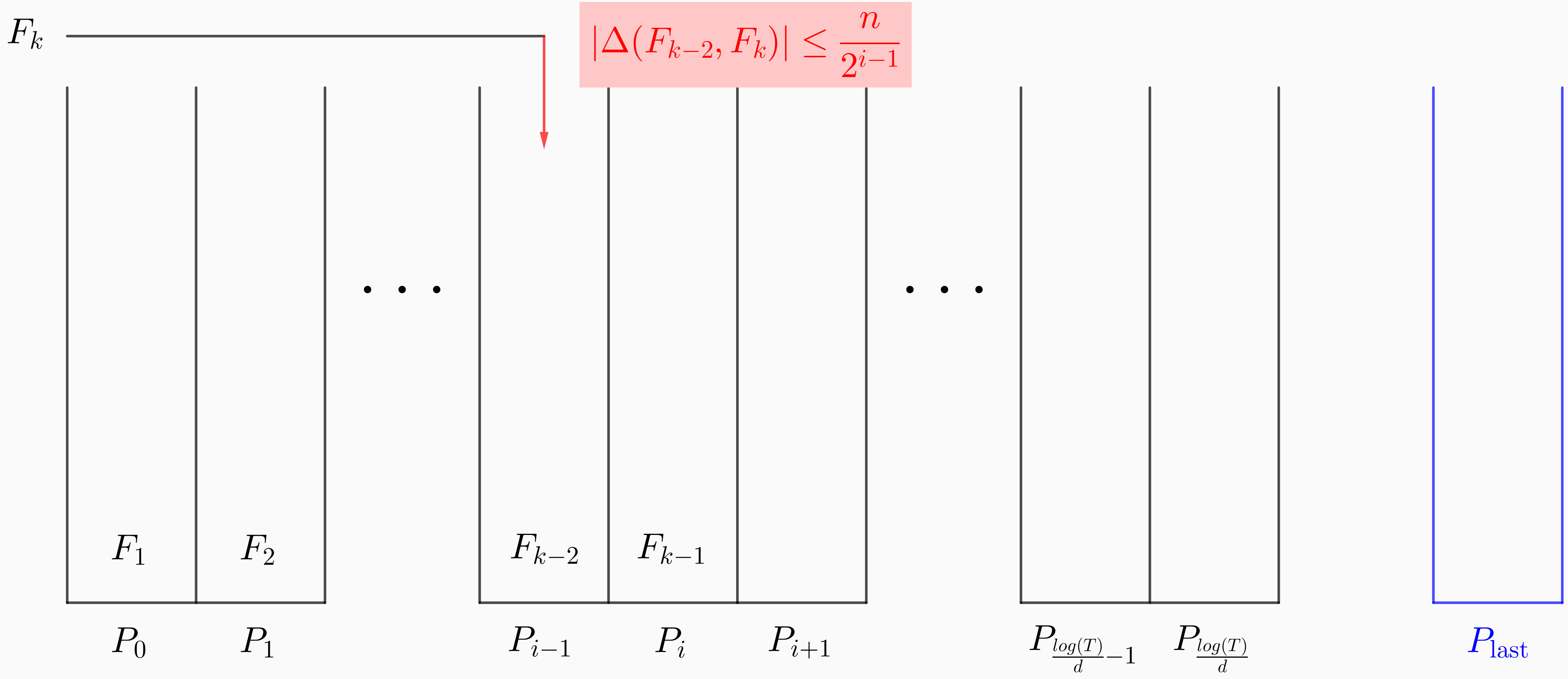

F_k
$|\Delta(F_2, F_k)| \leq \frac{n}{2}$
F_1
F_2
F_{k-2}
F_{k-1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration k :

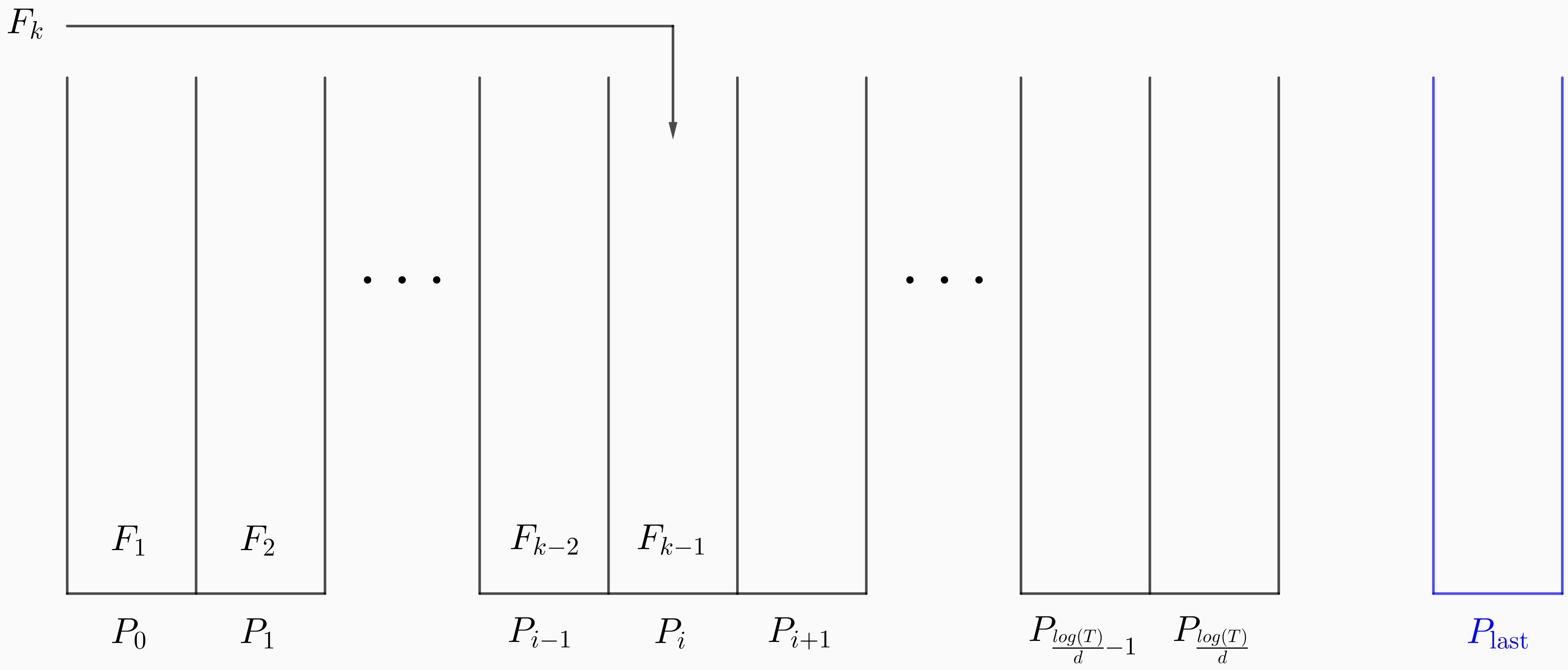


F_k
F_1
F_2
F_{k-2}
F_{k-1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

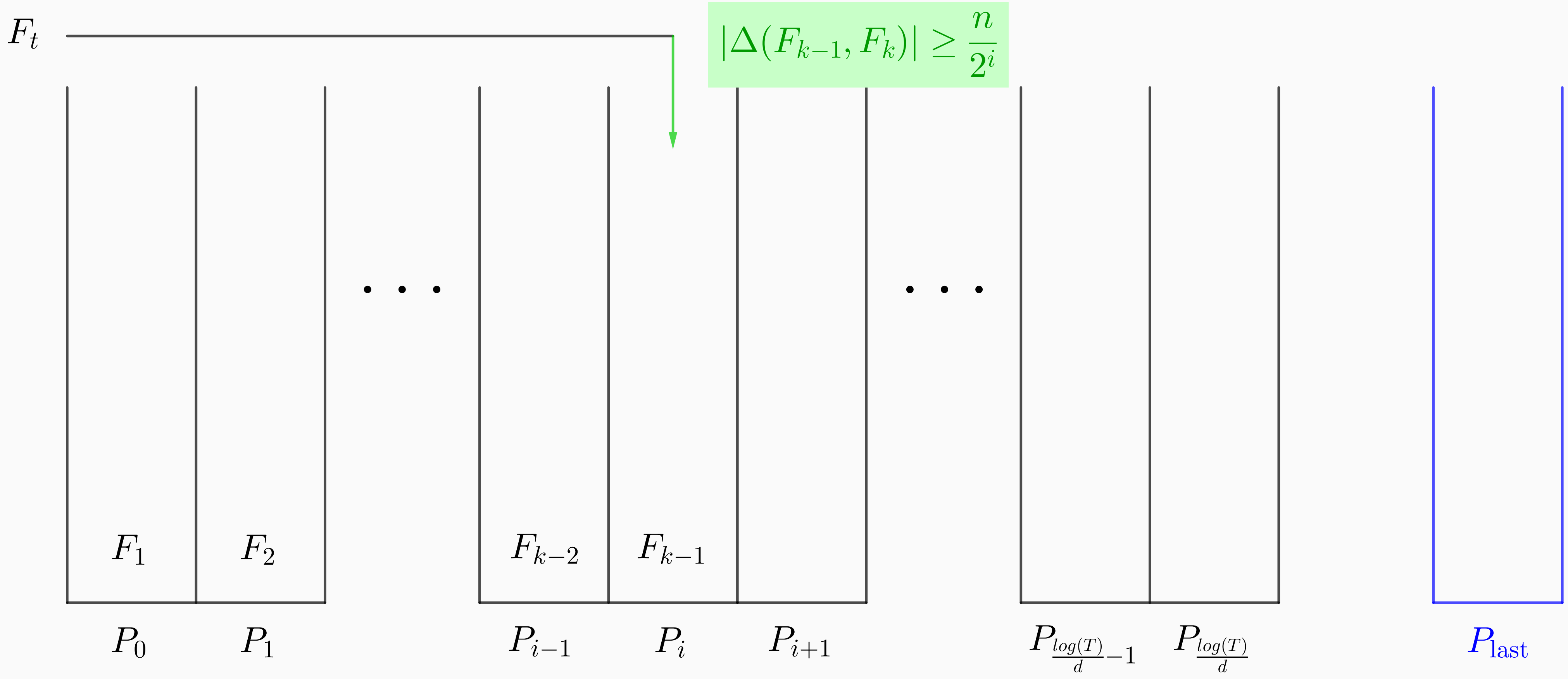
Iteration k :


F_k
$|\Delta(F_{k-2}, F_k)| \leq \frac{n}{2^{i-1}}$
F_1
F_2
F_{k-2}
F_{k-1}
$\dots$
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

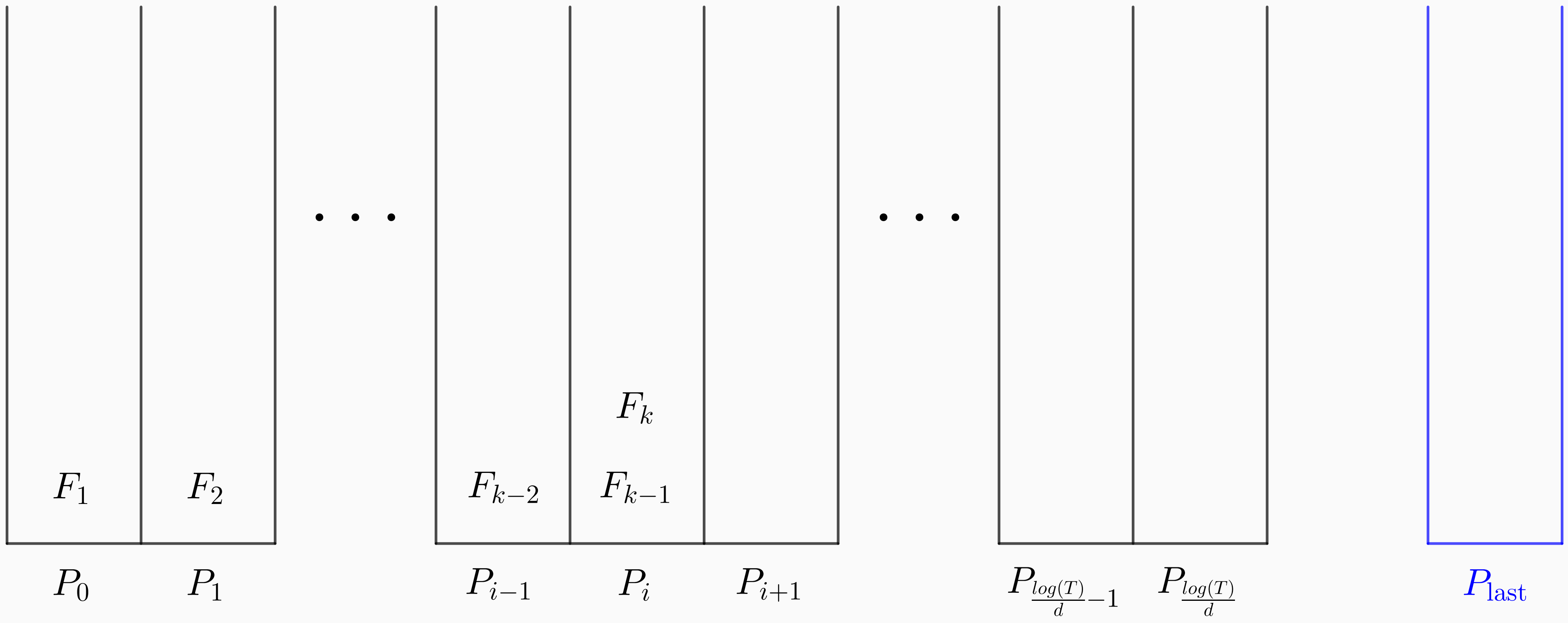
Iteration k :


F_k
F_1
F_2
F_{k-2}
F_{k-1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration k :

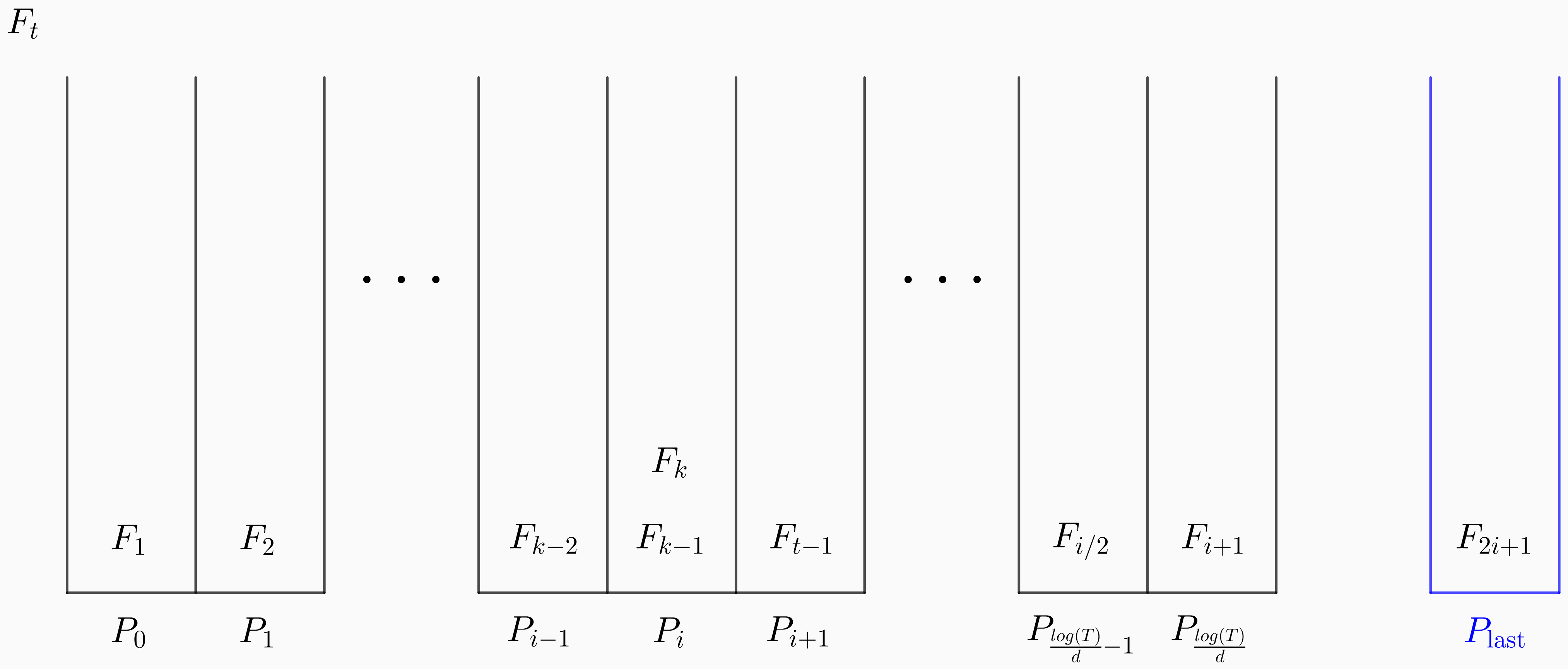

F_t
$|\Delta(F_{k-1}, F_k)| \geq \frac{n}{2^i}$
F_1
F_2
F_{k-2}
F_{k-1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration k :

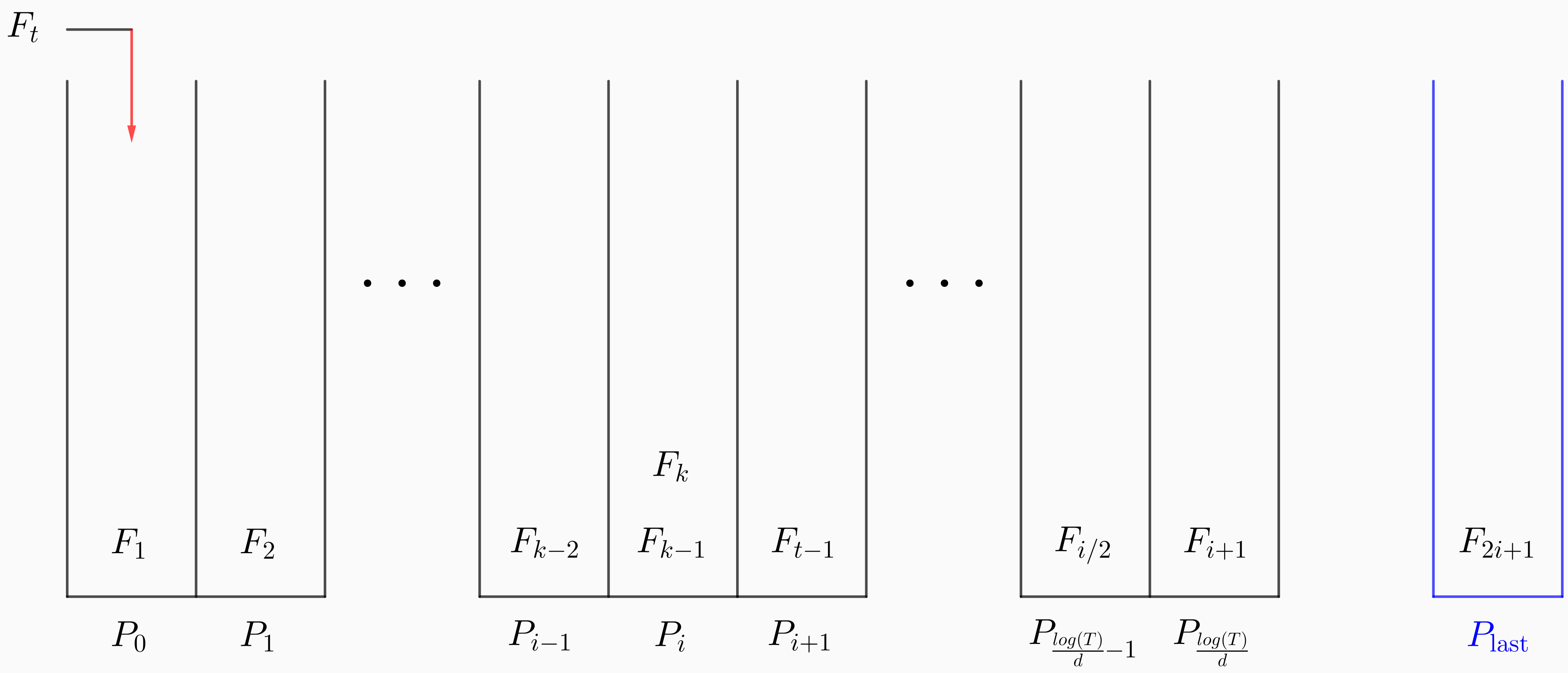

F_1
F_2
$\dots$
F_{k-2}
F_k
F_{k-1}
$\dots$
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Overview of the proof (construction)

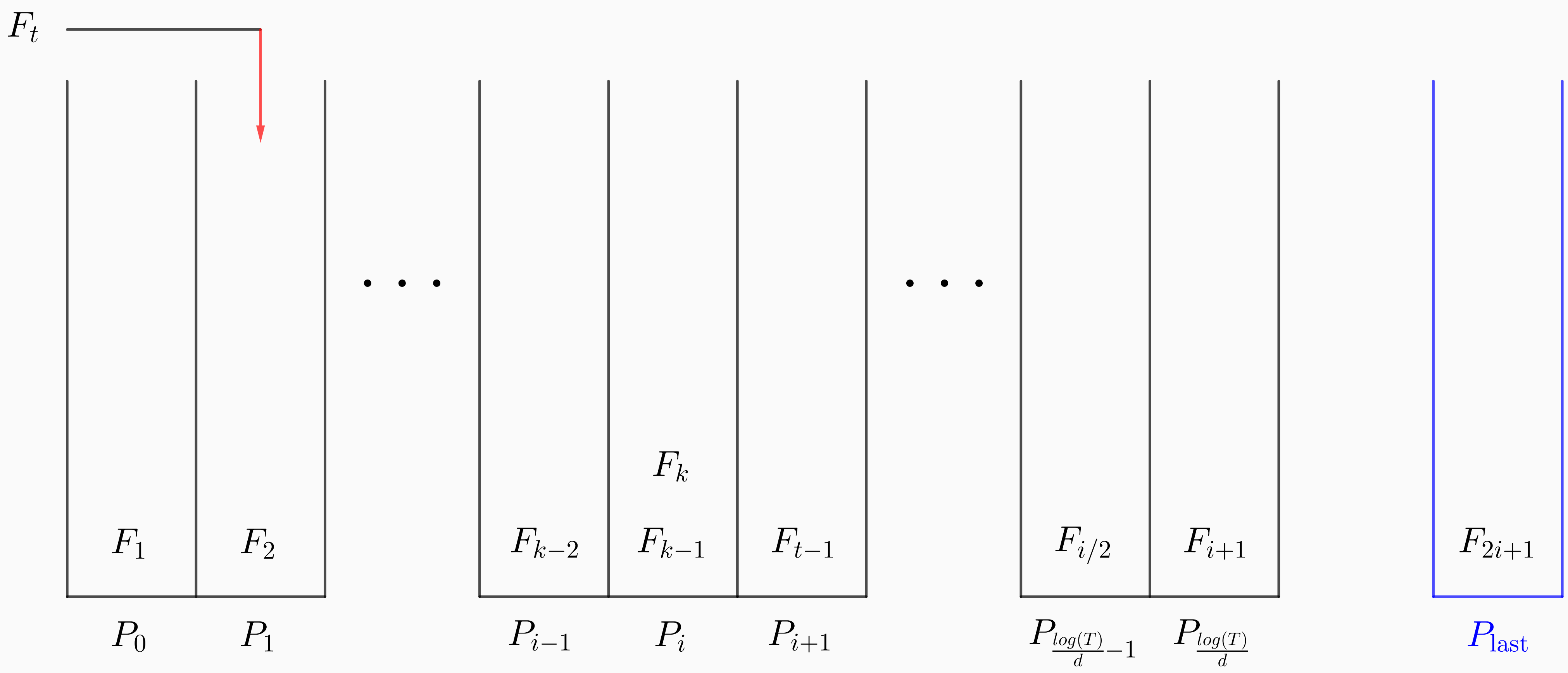
Iteration t :



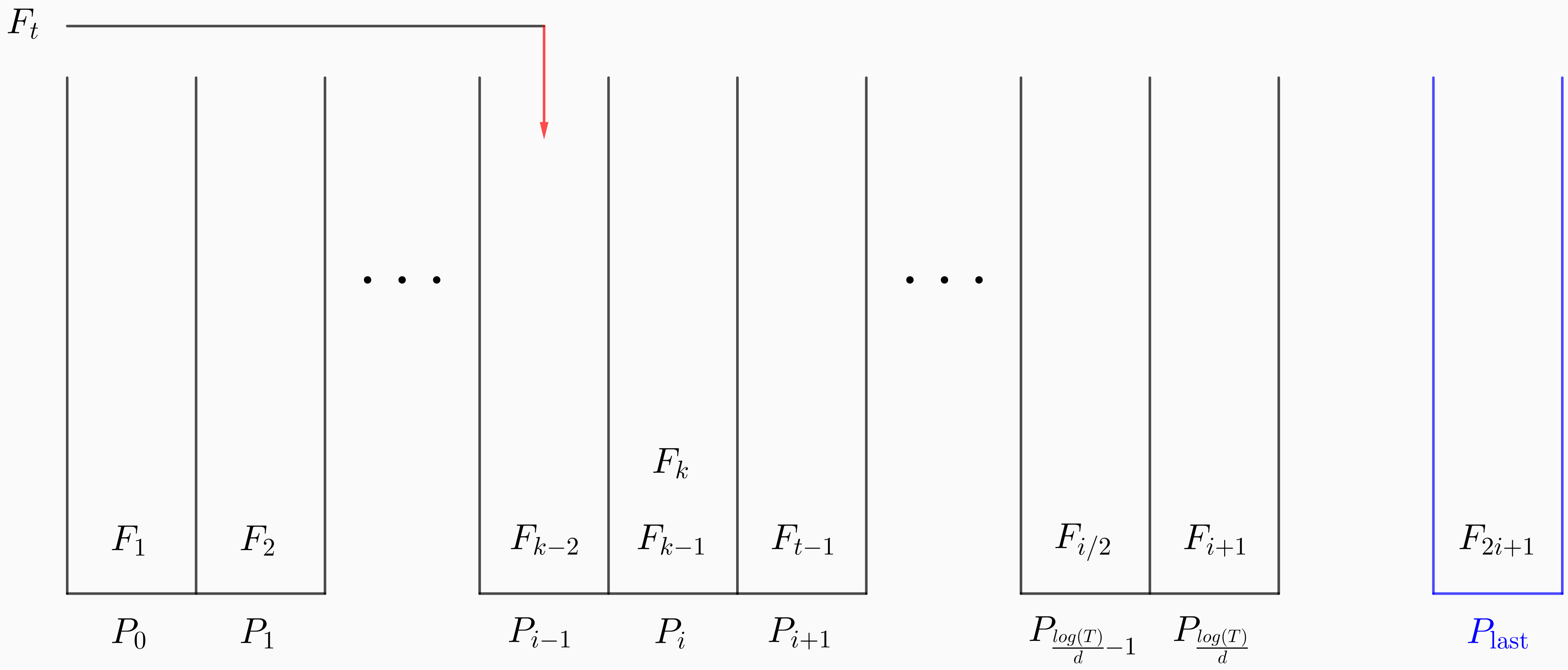
Iteration t :


F_t
F_1
F_2
F_{k-2}
F_k
F_{k-1}
F_{t-1}
$F_{i/2}$
F_{i+1}
F_{2i+1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

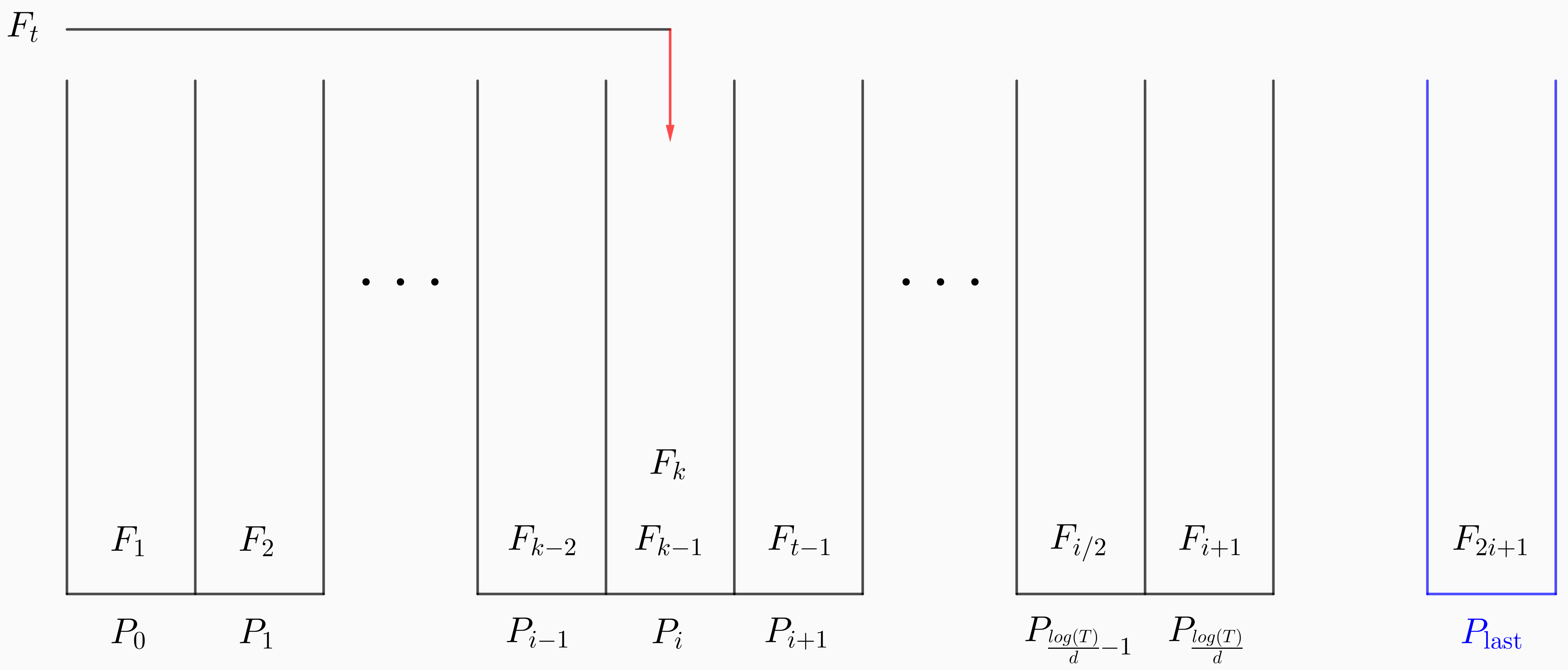
Iteration t :


F_t
$\dots$
$\dots$
F_k
F_1
F_2
F_{k-2}
F_{k-1}
F_{t-1}
$F_{i/2}$
F_{i+1}
F_{2i+1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

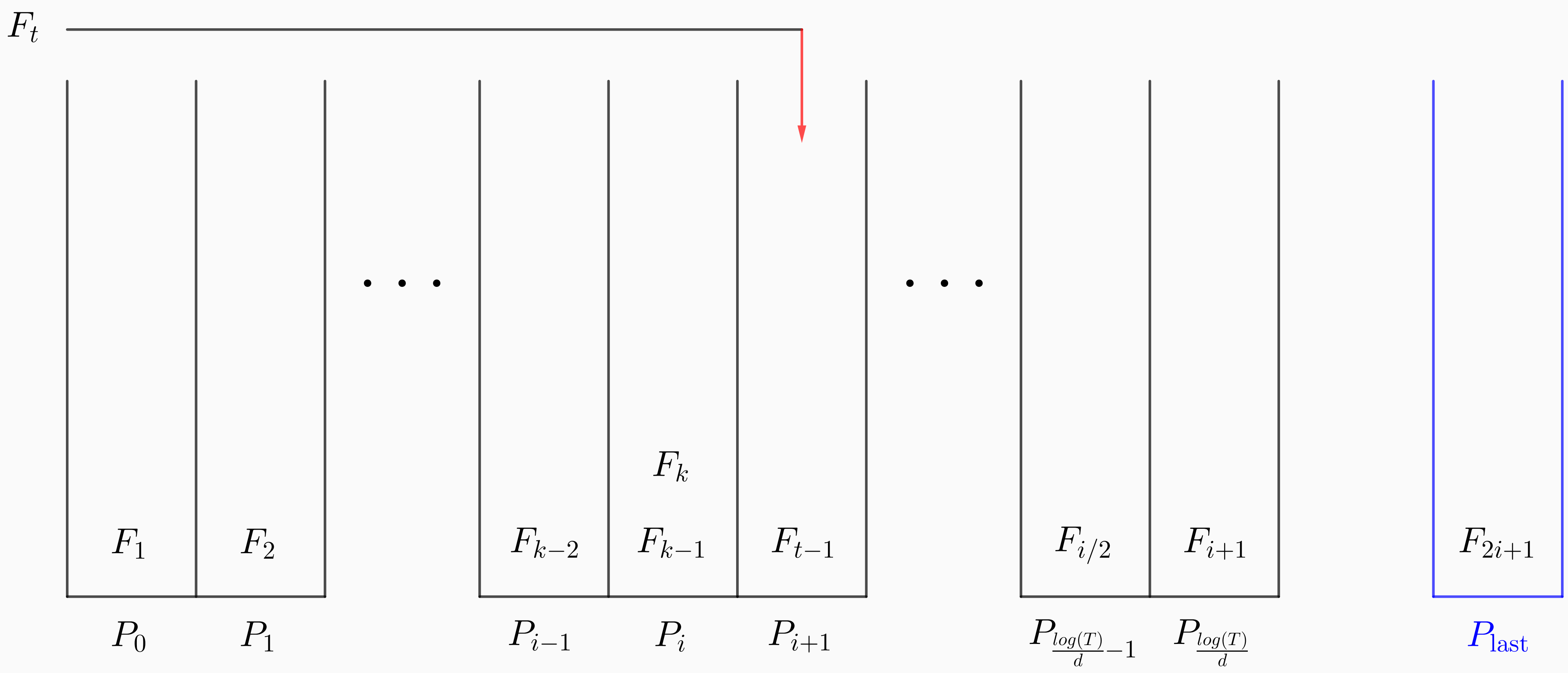
Iteration t :


F_t
F_1
F_2
F_{k-2}
F_k
F_{k-1}
F_{t-1}
$F_{i/2}$
F_{i+1}
F_{2i+1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

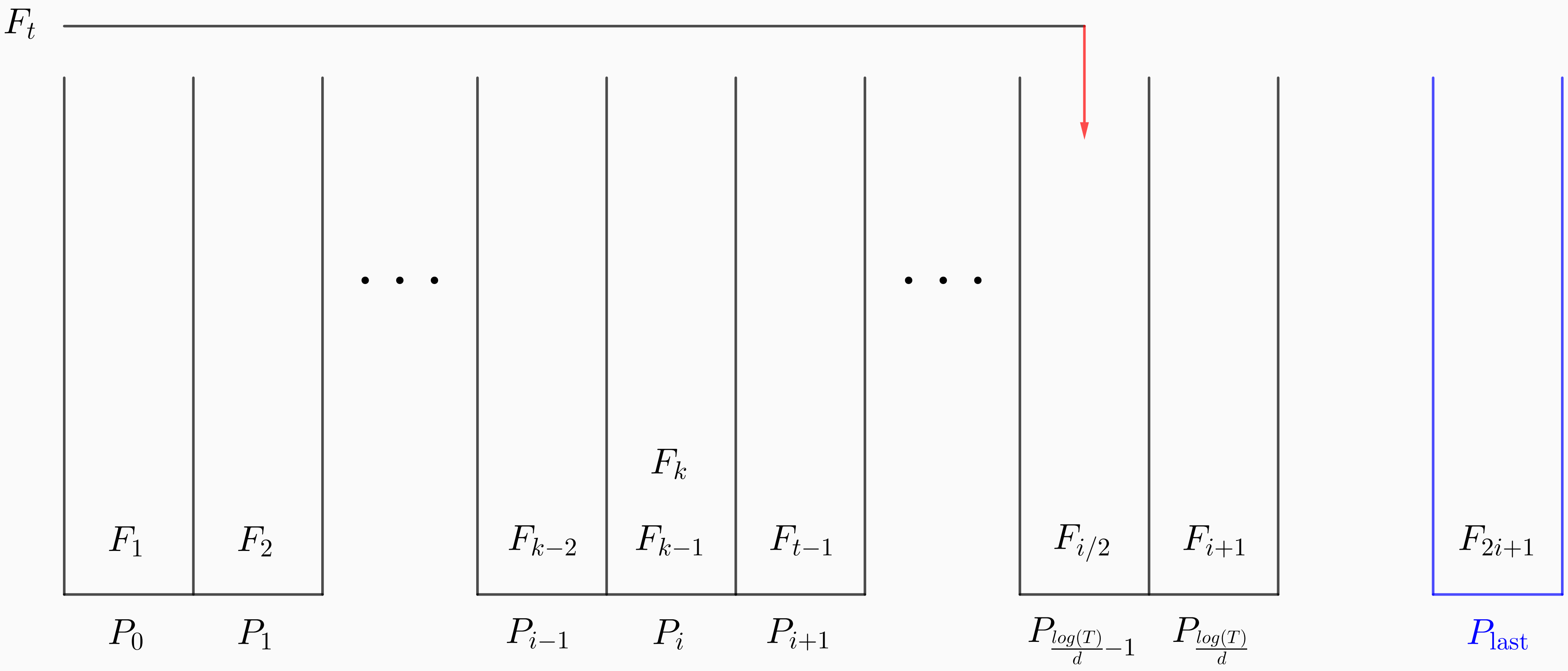
Iteration t :


F_t
F_k
F_1
F_2
F_{k-2}
F_{k-1}
F_{t-1}
$F_{i/2}$
F_{i+1}
F_{2i+1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

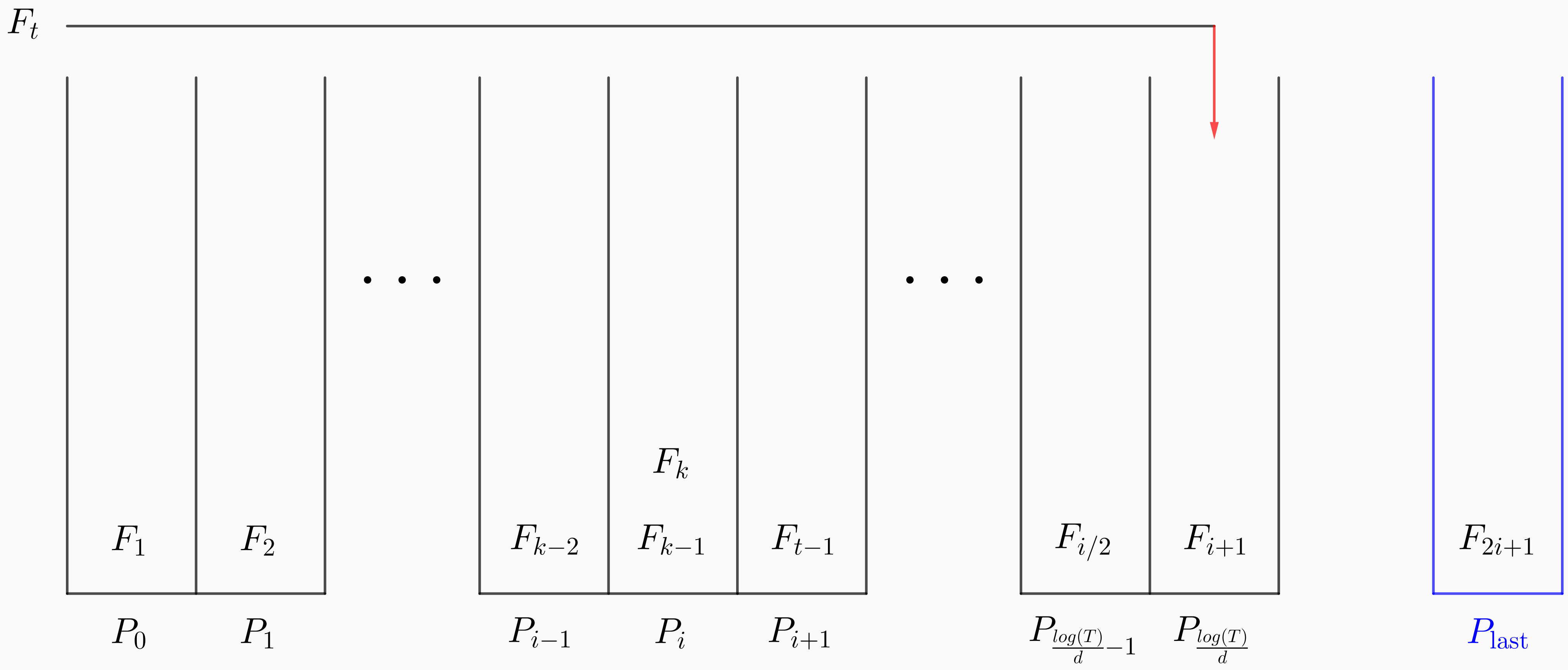
Iteration t :


F_t
$\cdots$
$\cdots$
F_k
F_1
F_2
F_{k-2}
F_{k-1}
F_{t-1}
$F_{i/2}$
F_{i+1}
F_{2i+1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration t :

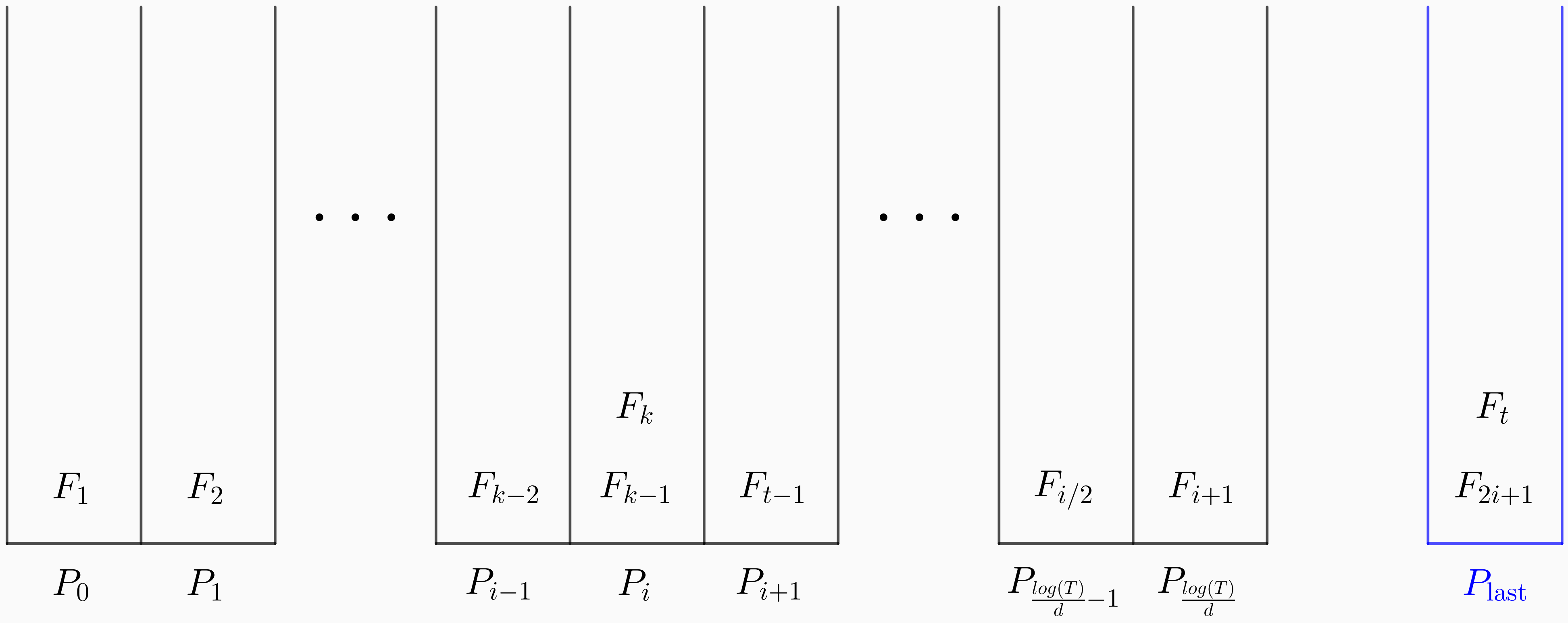

F_t
$\dots$
$\dots$
F_k
F_1
F_2
F_{k-2}
F_{k-1}
F_{t-1}
$F_{i/2}$
F_{i+1}
F_{2i+1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Iteration t :


F_t
F_k
F_1
F_2
F_{k-2}
F_{k-1}
F_{t-1}
$F_{i/2}$
F_{i+1}
F_{2i+1}
P_0
P_1
P_{i-1}
P_i
P_{i+1}
$P_{\frac{log(T)}{d}-1}$
$P_{\frac{log(T)}{d}}$
P_{last}

Overview of the proof (construction)

Iteration t :



Overview of the proof (bound)

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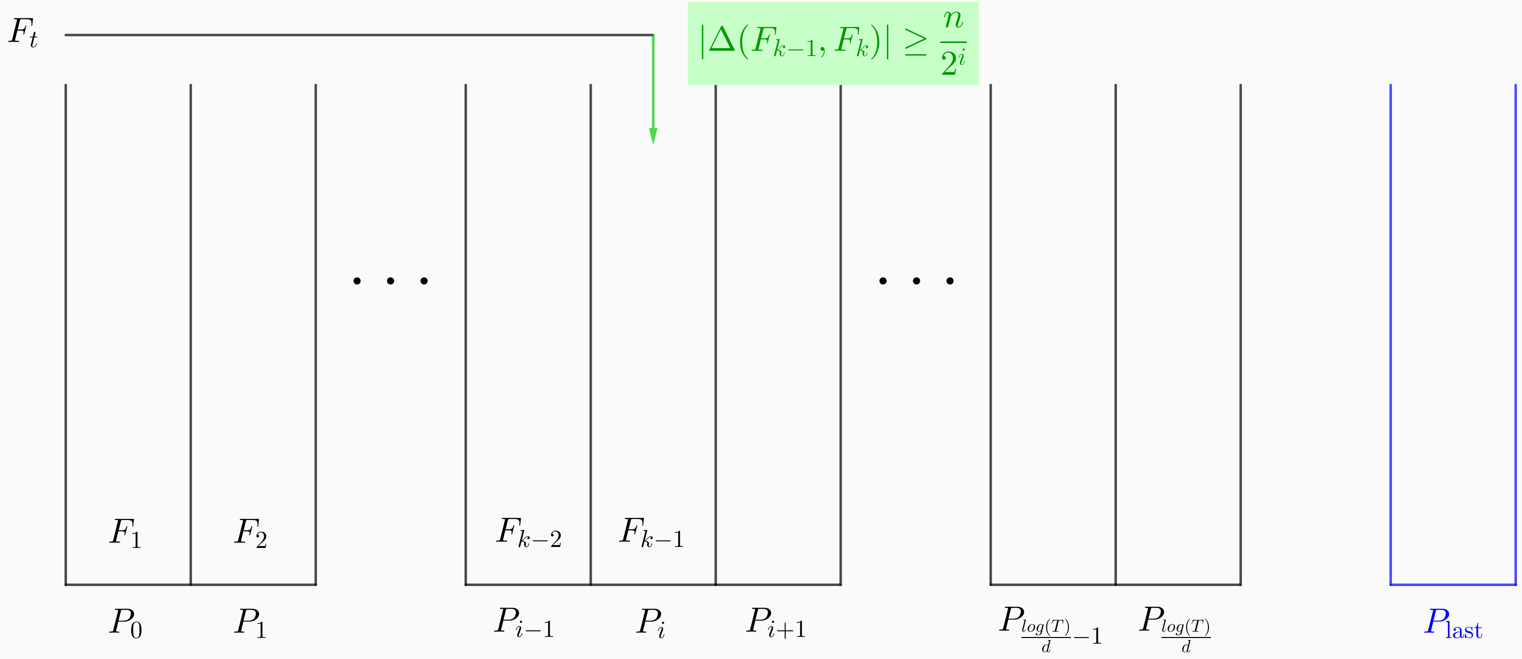
Iteration k :



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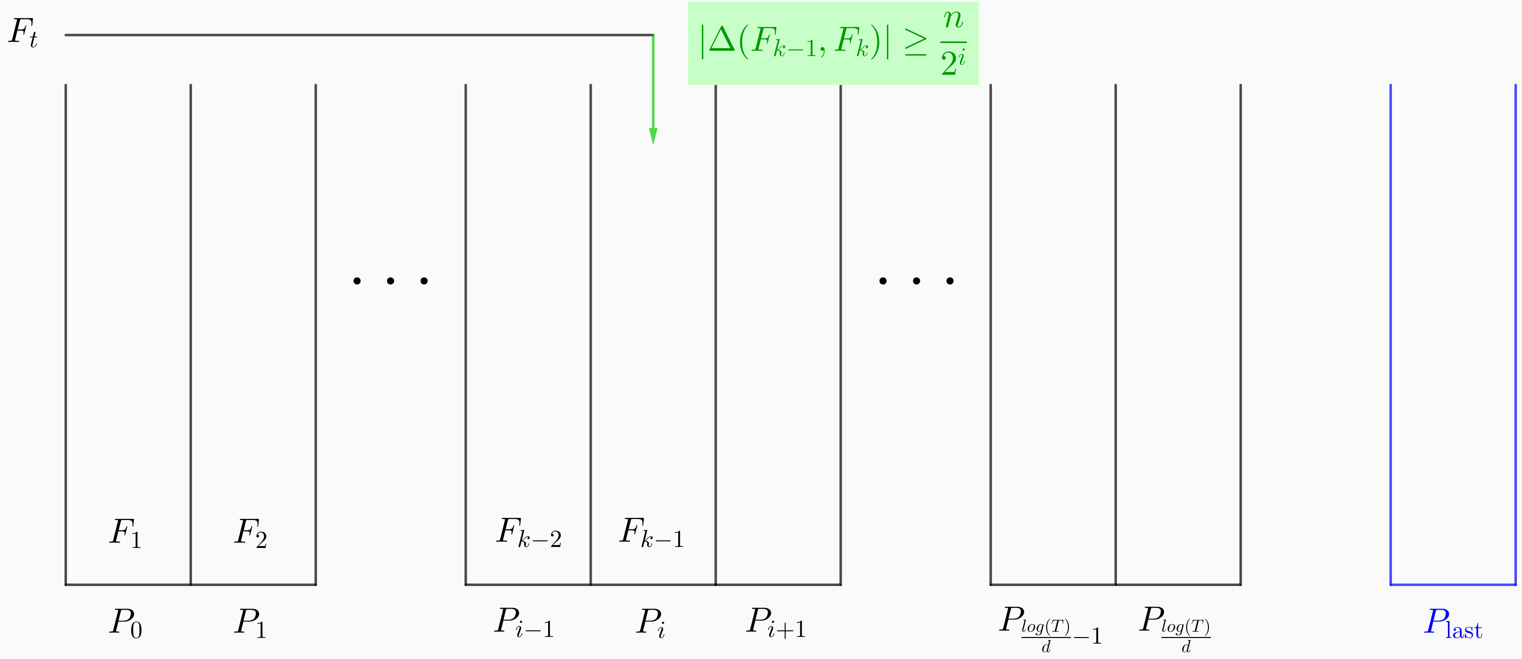

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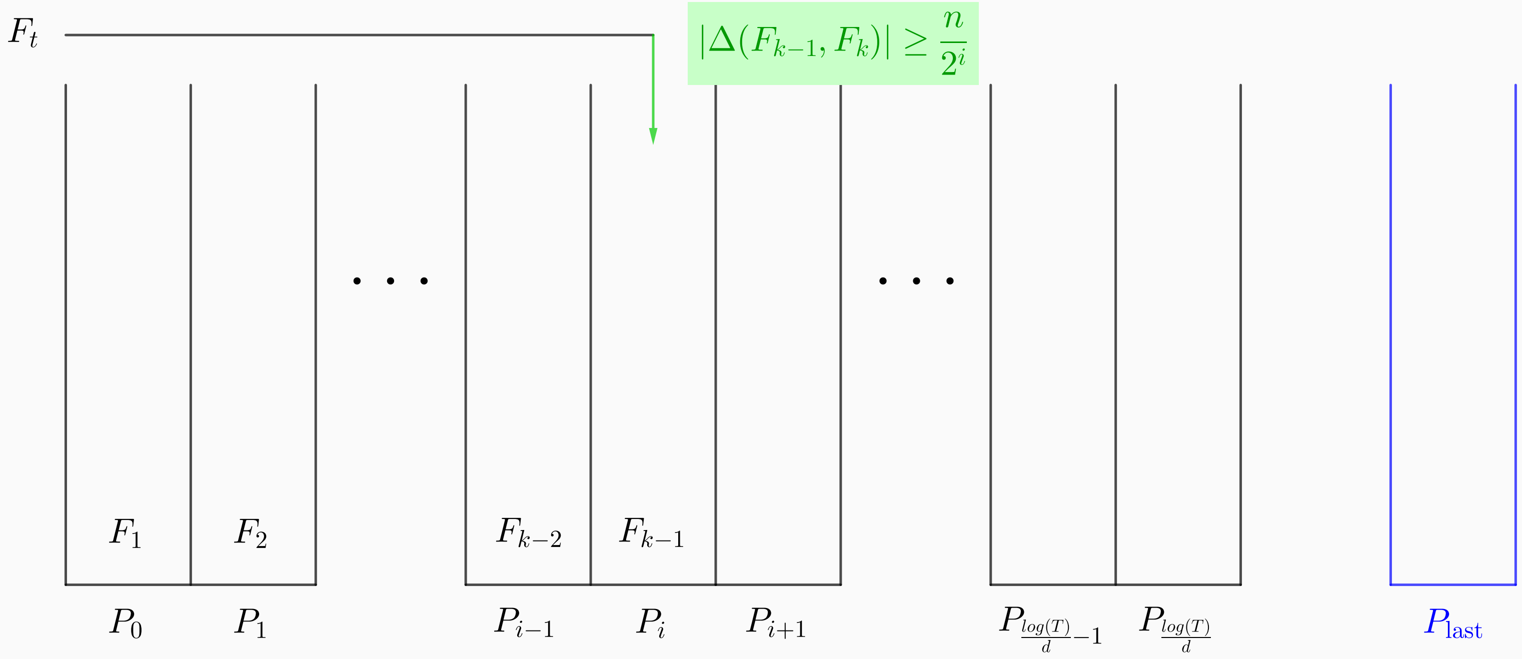

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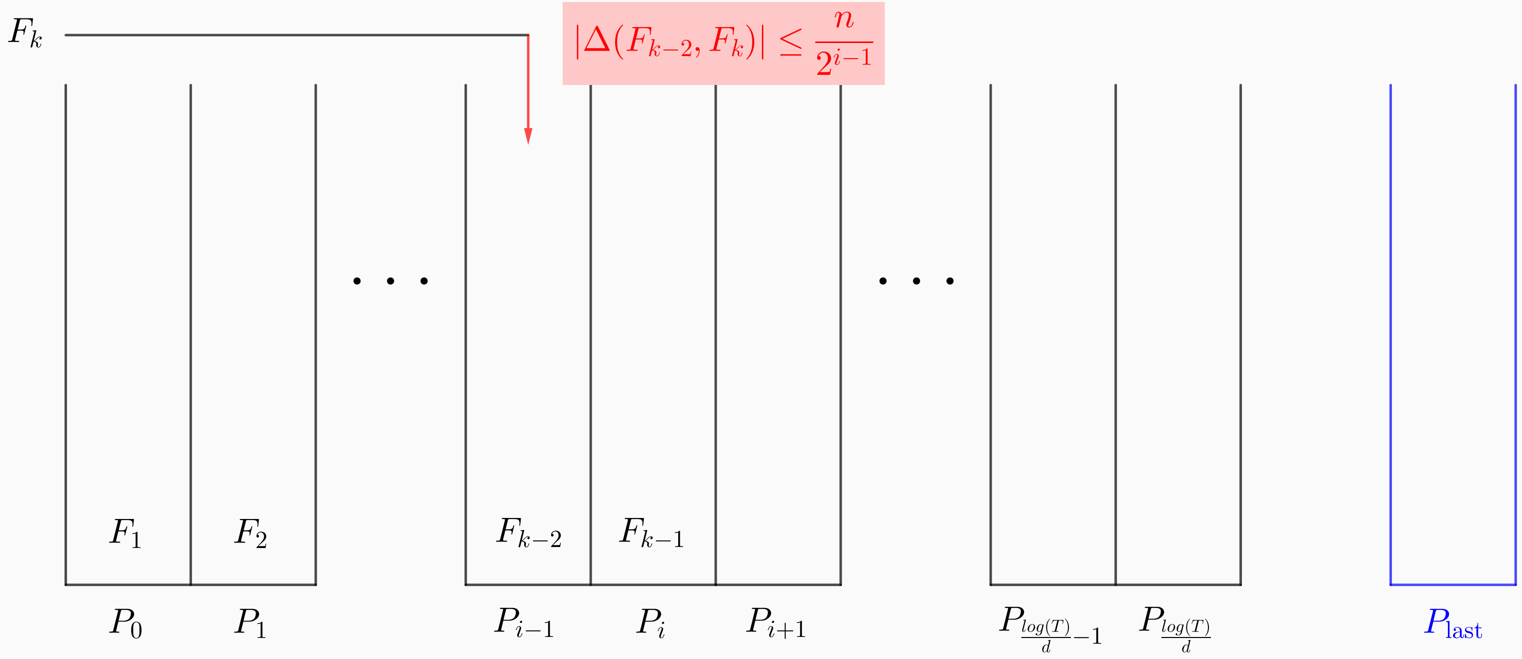

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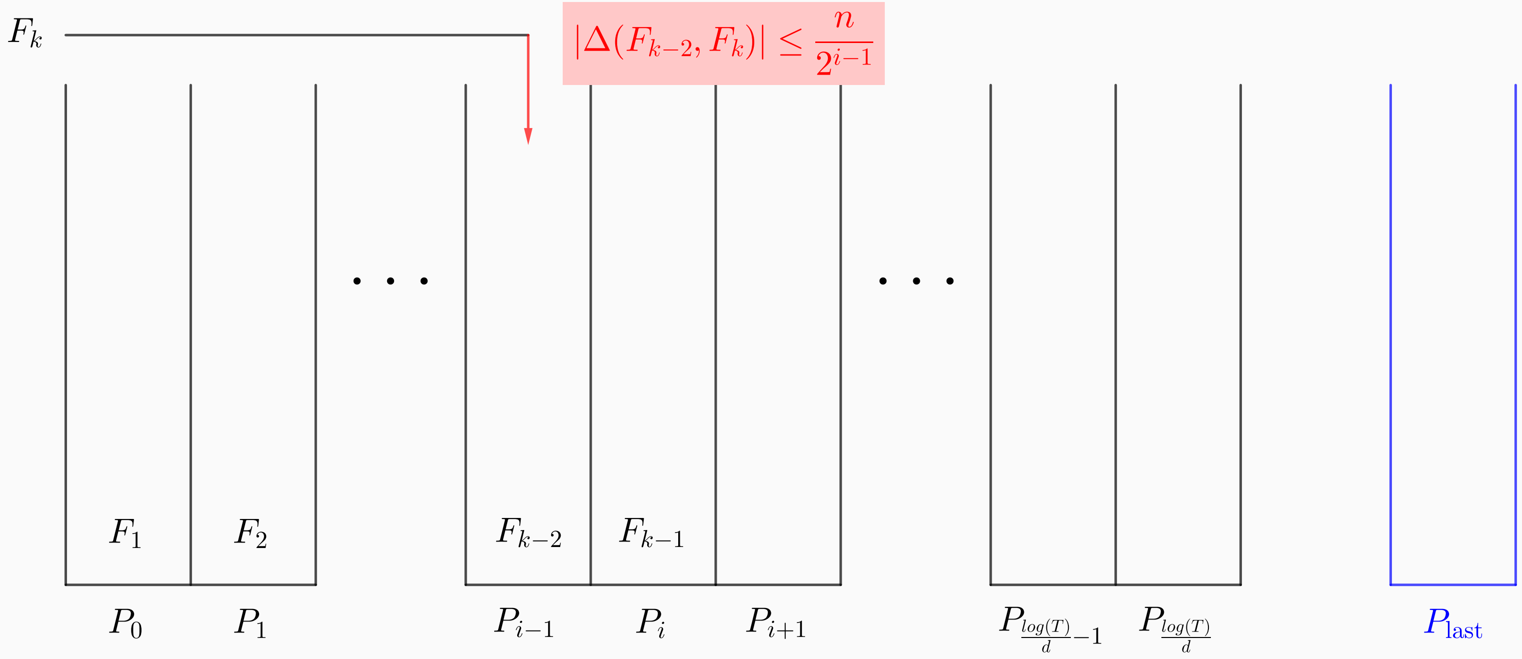

Discrepancy and symmetric difference

Symmetric difference: $\Delta(F_1, F_2) = (F_1 \cup F_2) \setminus (F_1 \cap F_2)$

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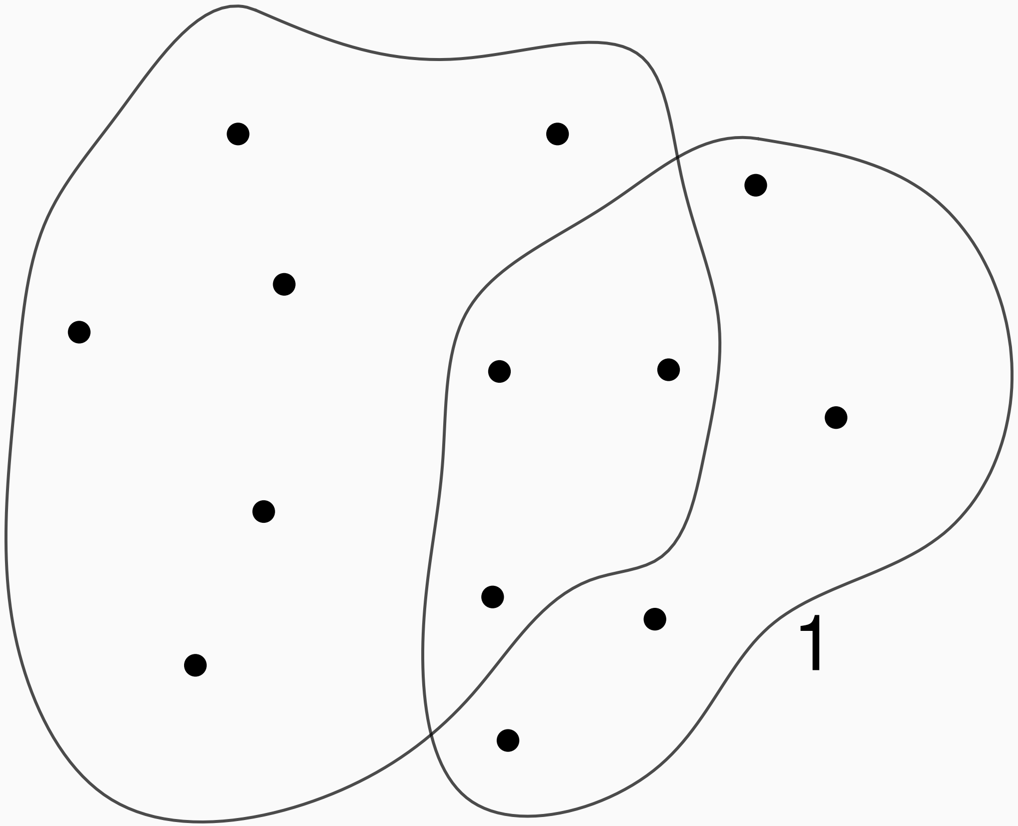

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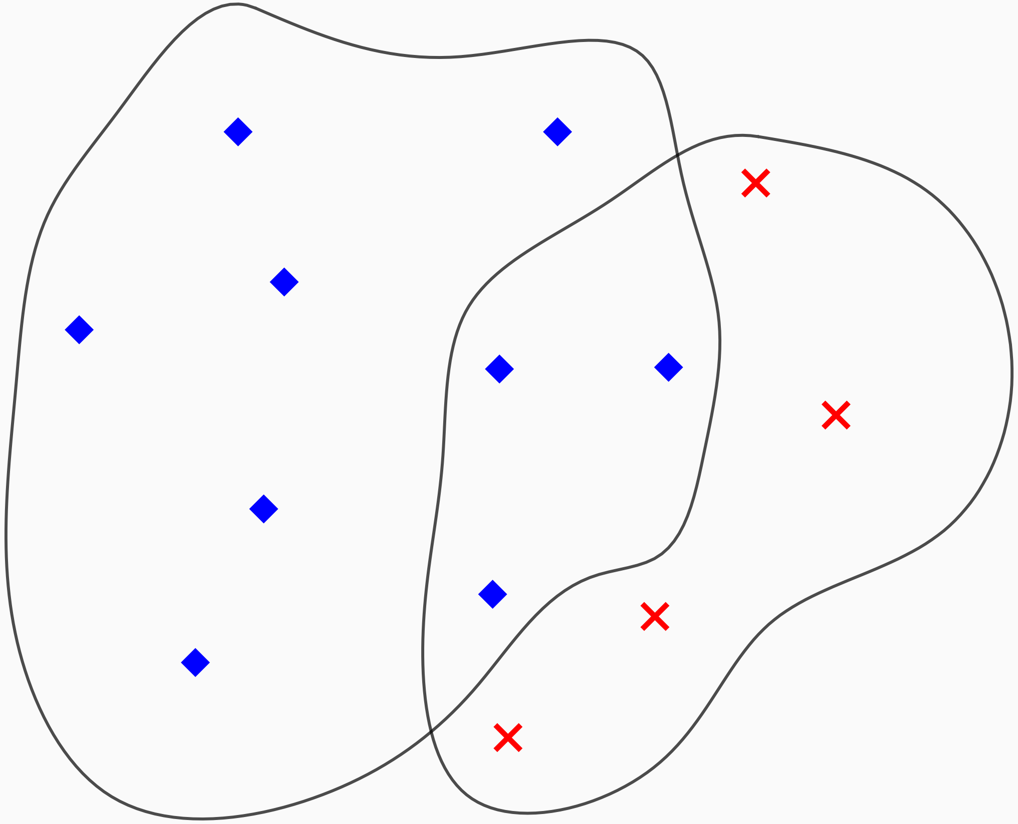

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Low-crossing partitions

Definition

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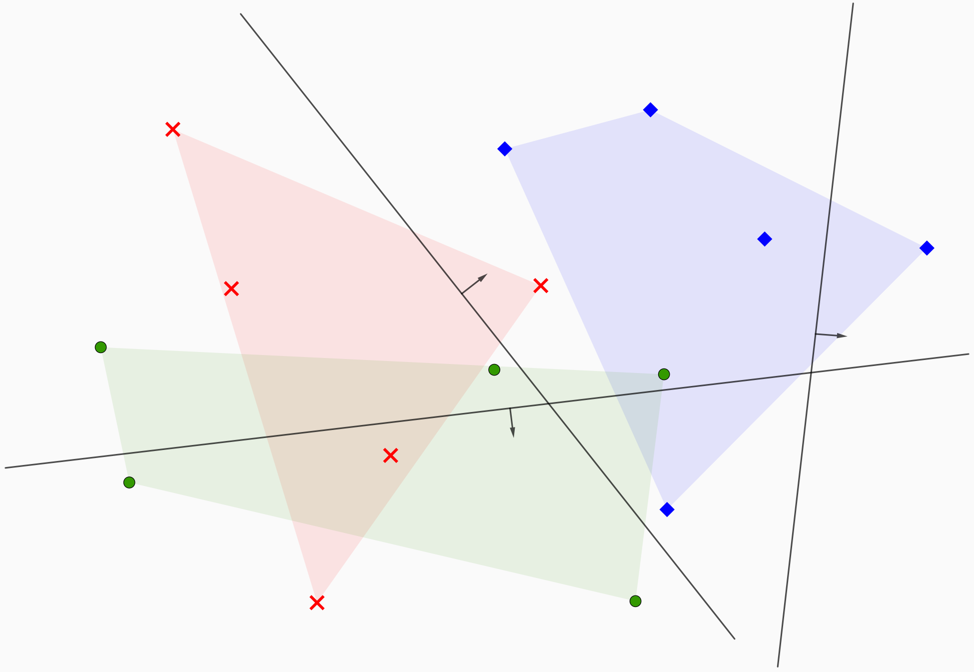

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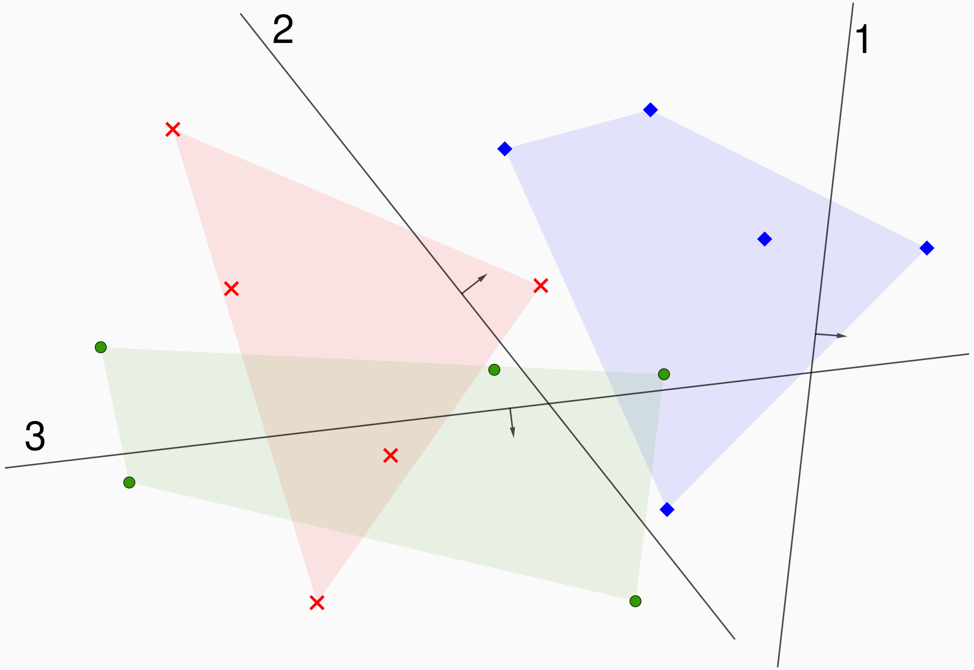

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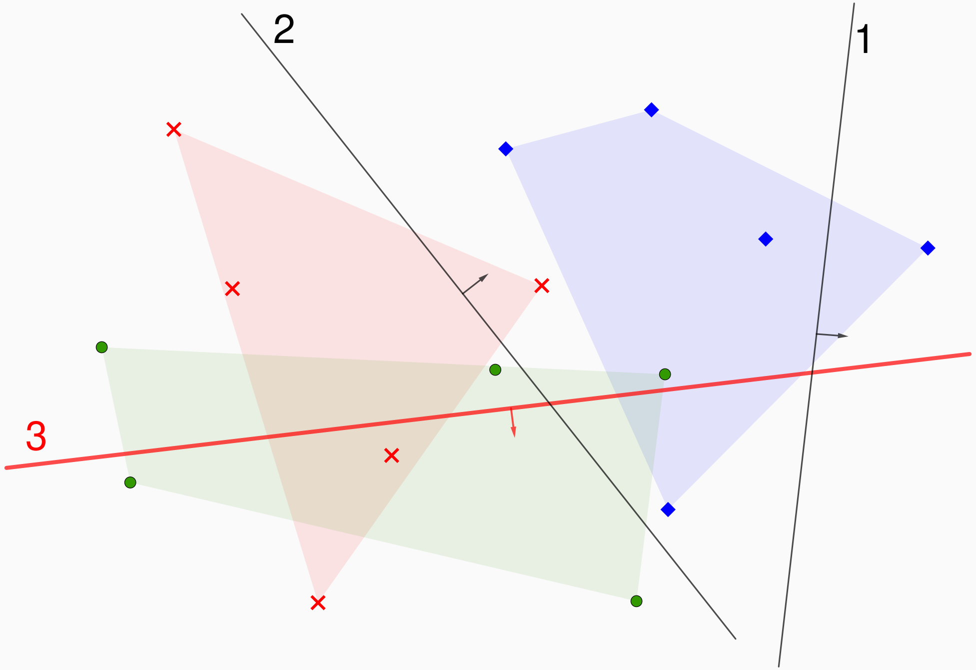

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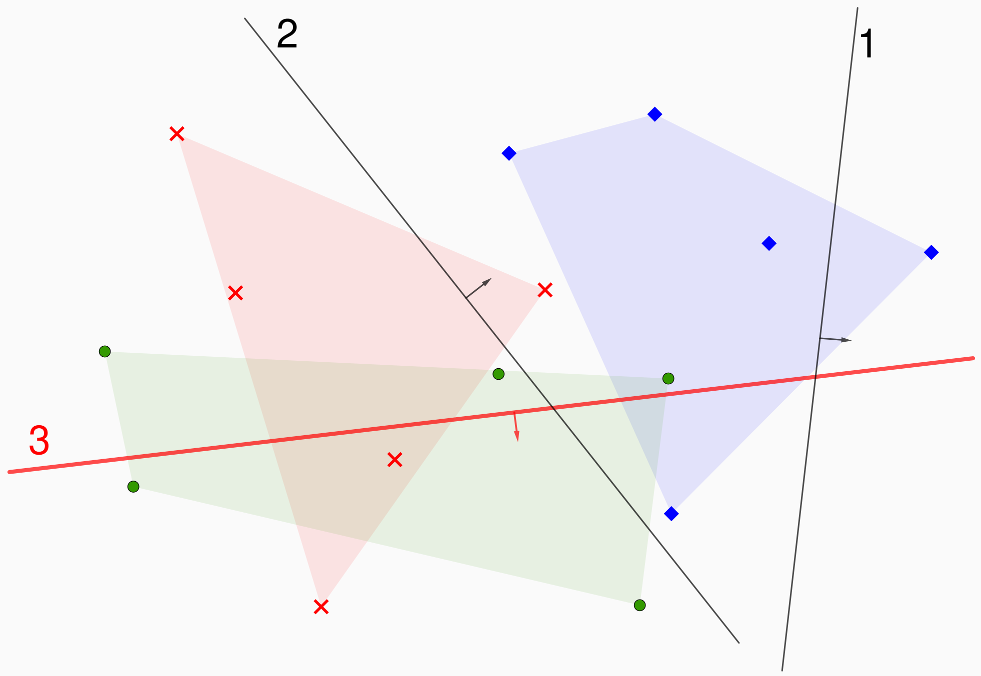

Goal: Compute a small crossing number partition

Previous work

- **Matoušek (1992)**: $O\left(t^{1-\frac{1}{d}}\right)$ for halfspaces in $\mathbb{R}^d$.

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Only halfspaces in low dimension.

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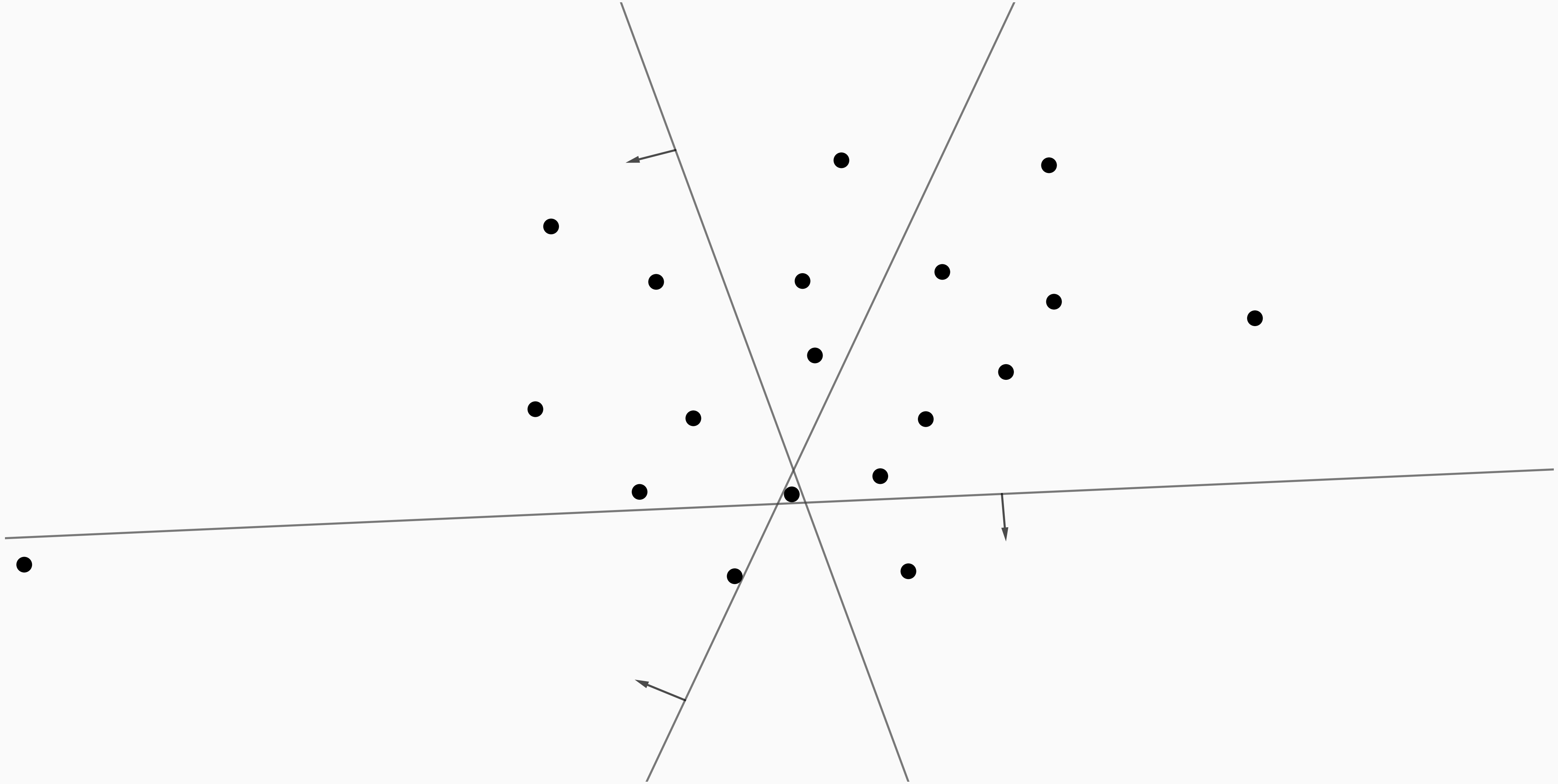
$\Rightarrow$ Potential function to greedily construct low-crossing partitions.

A greedy algorithm



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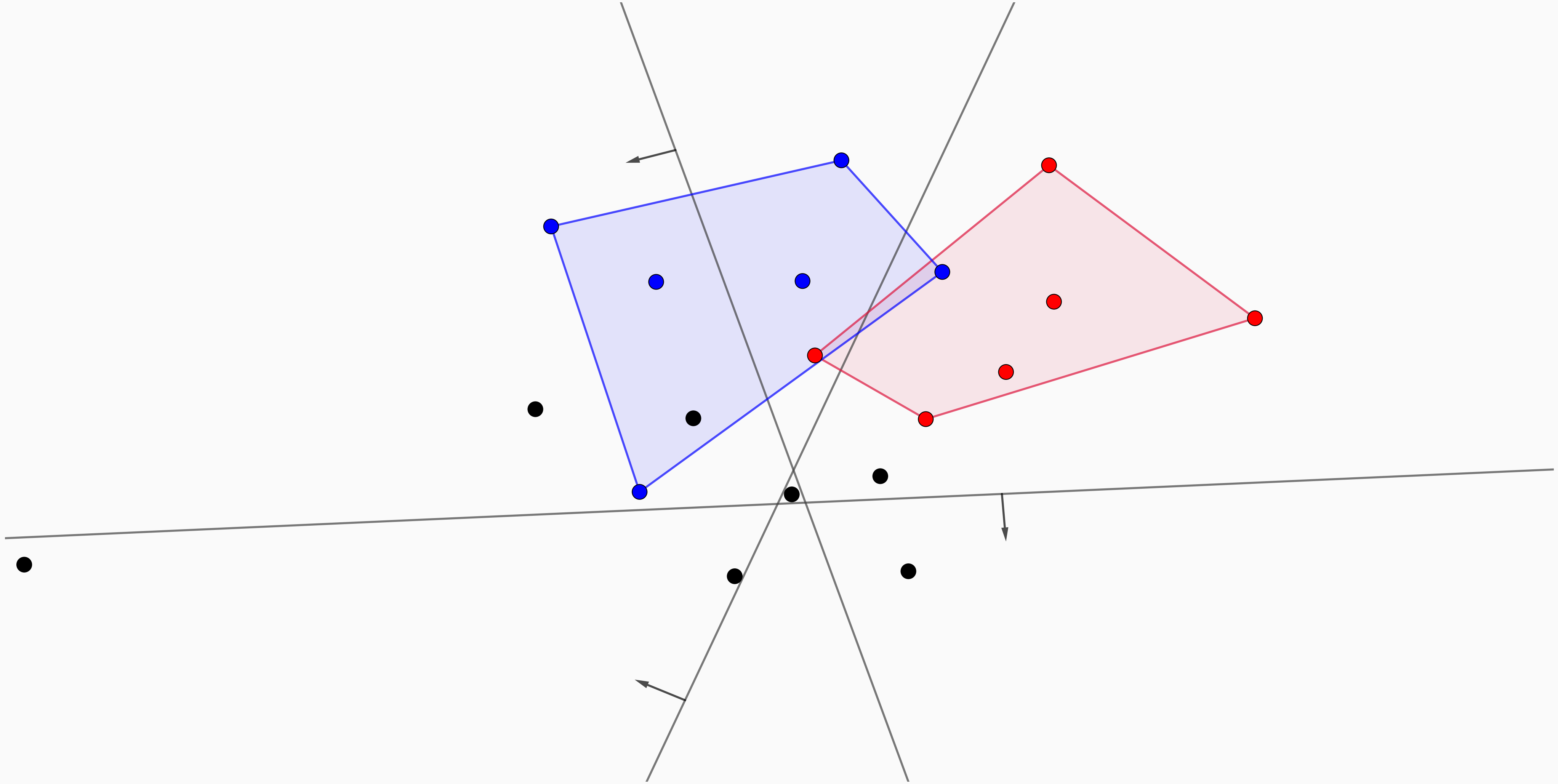

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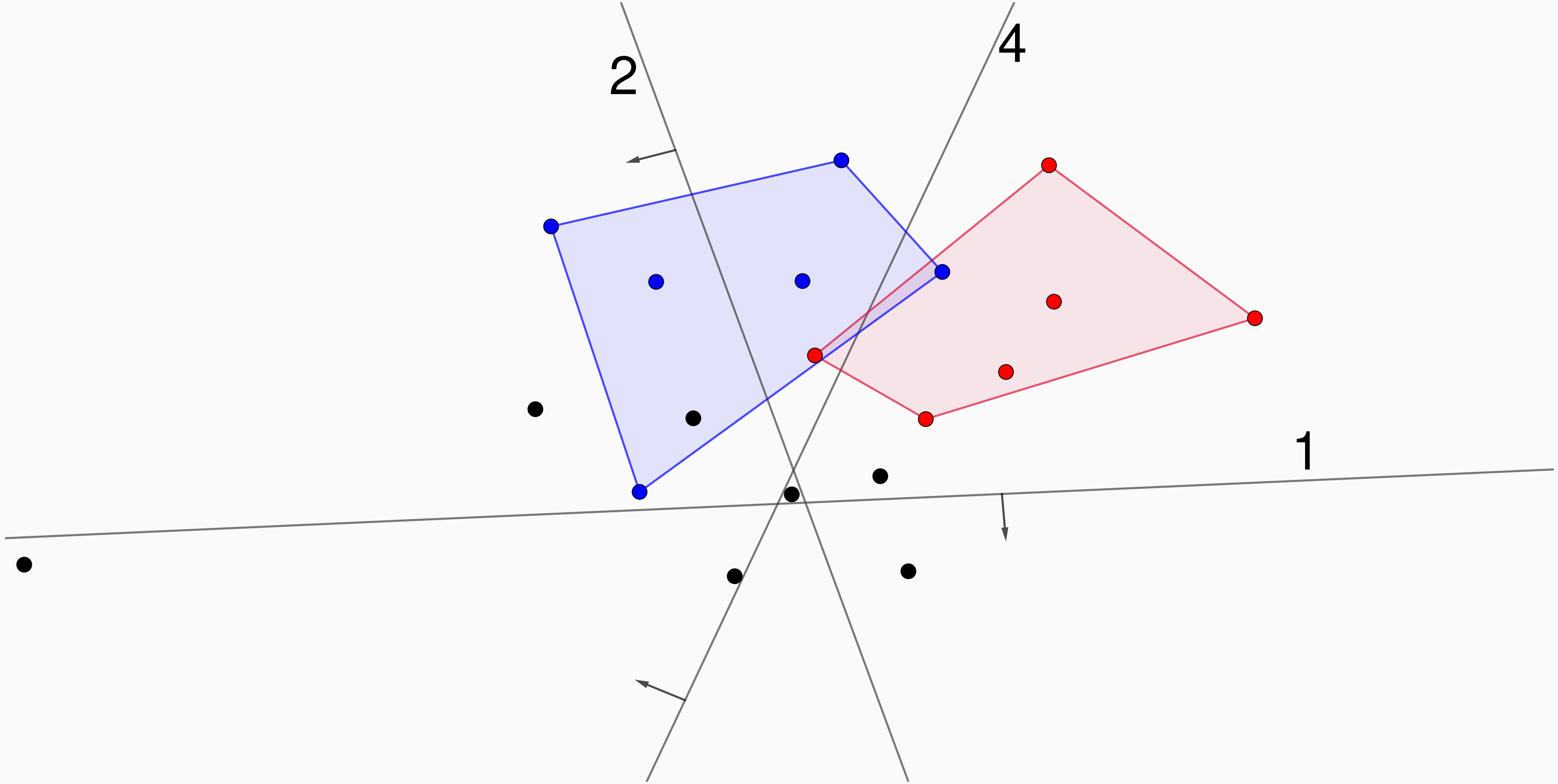

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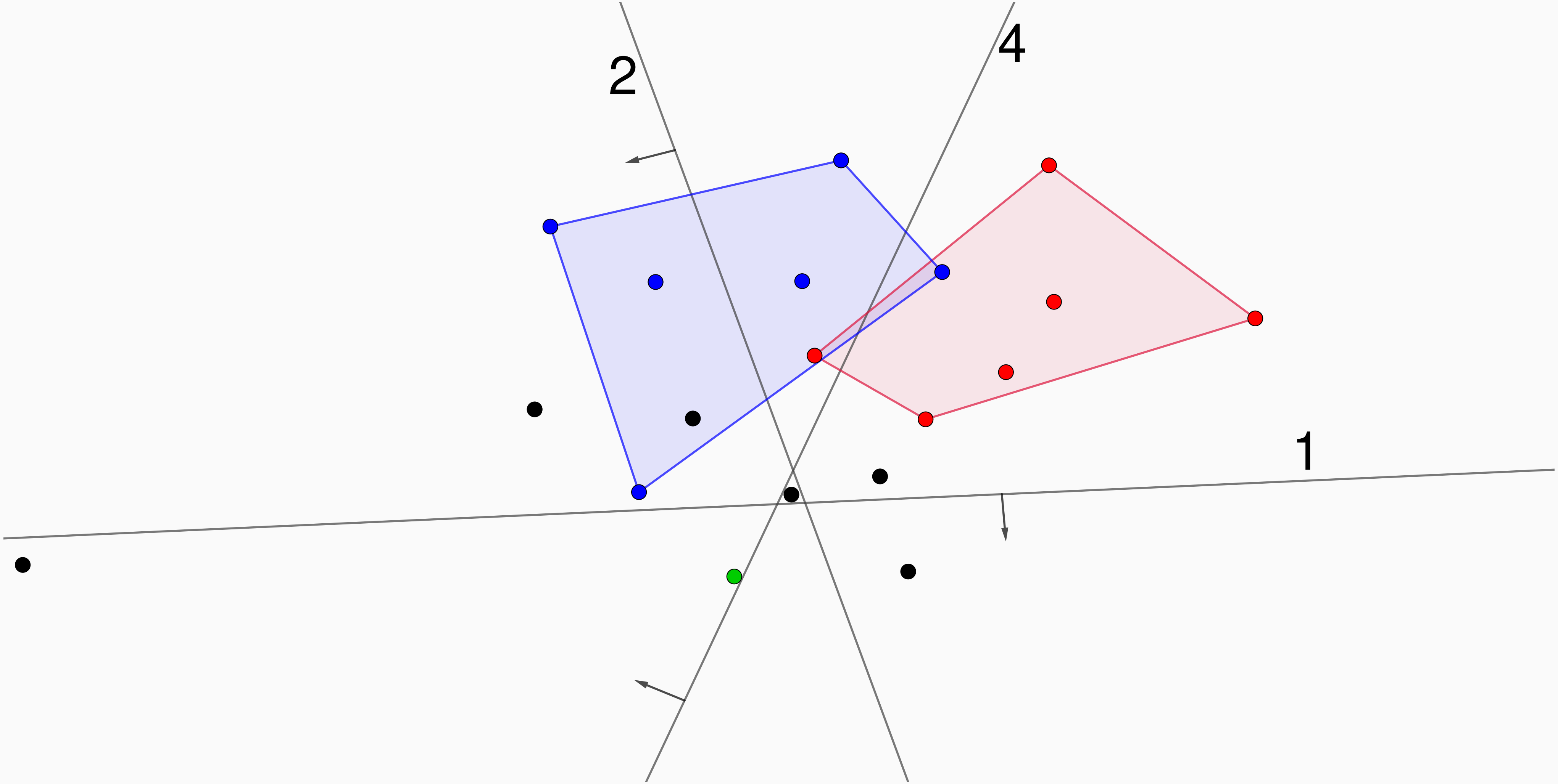

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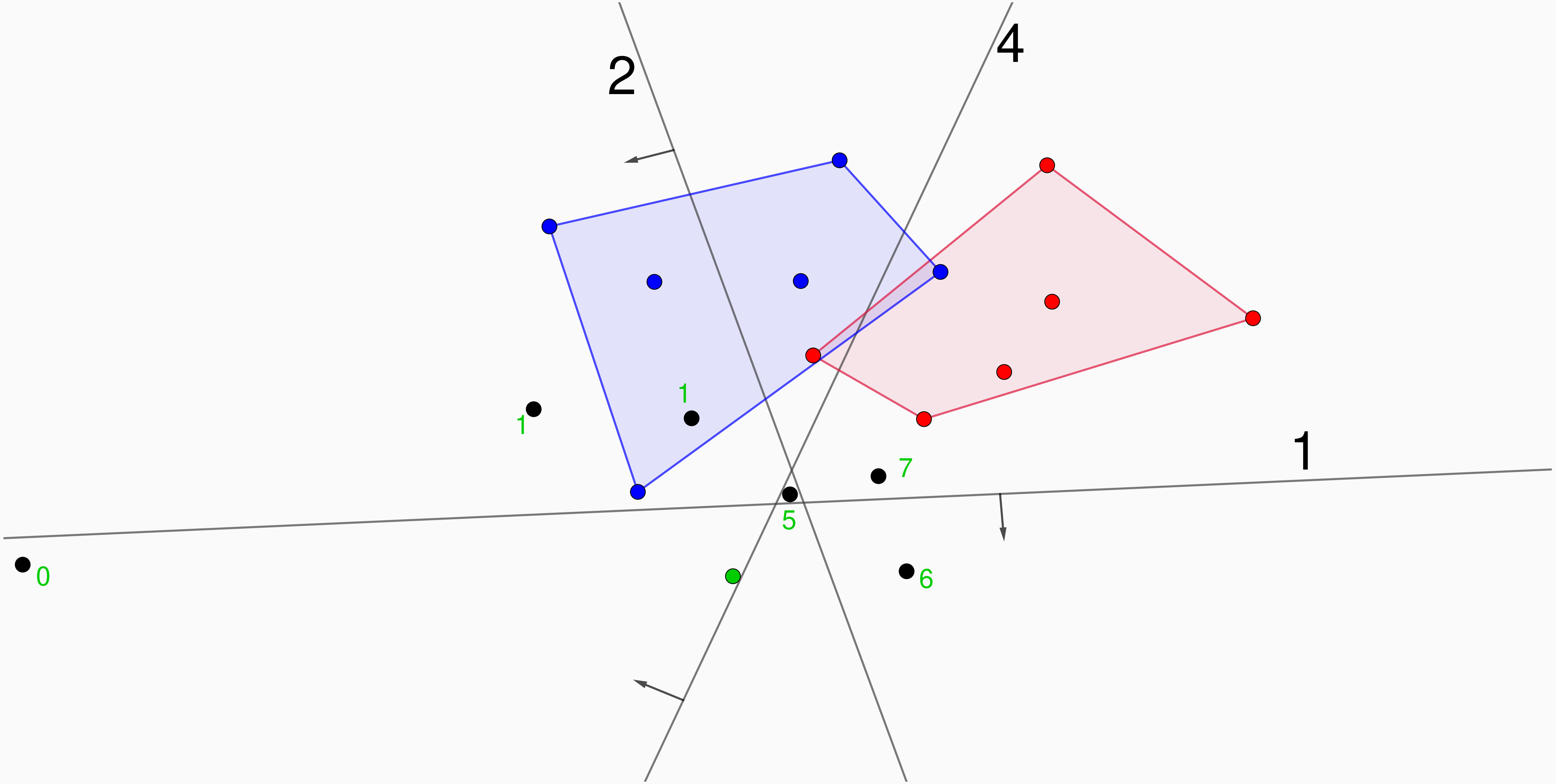

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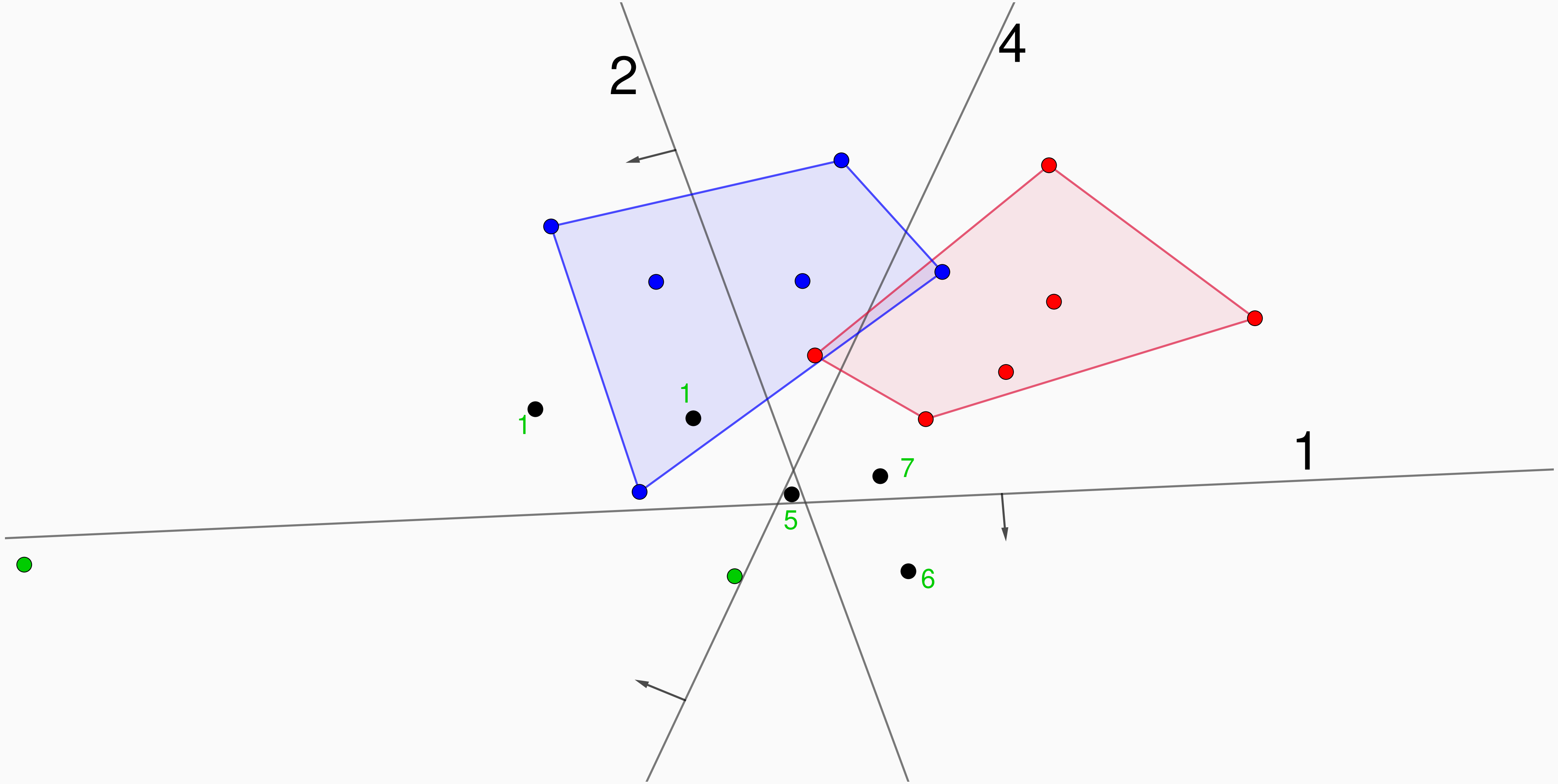
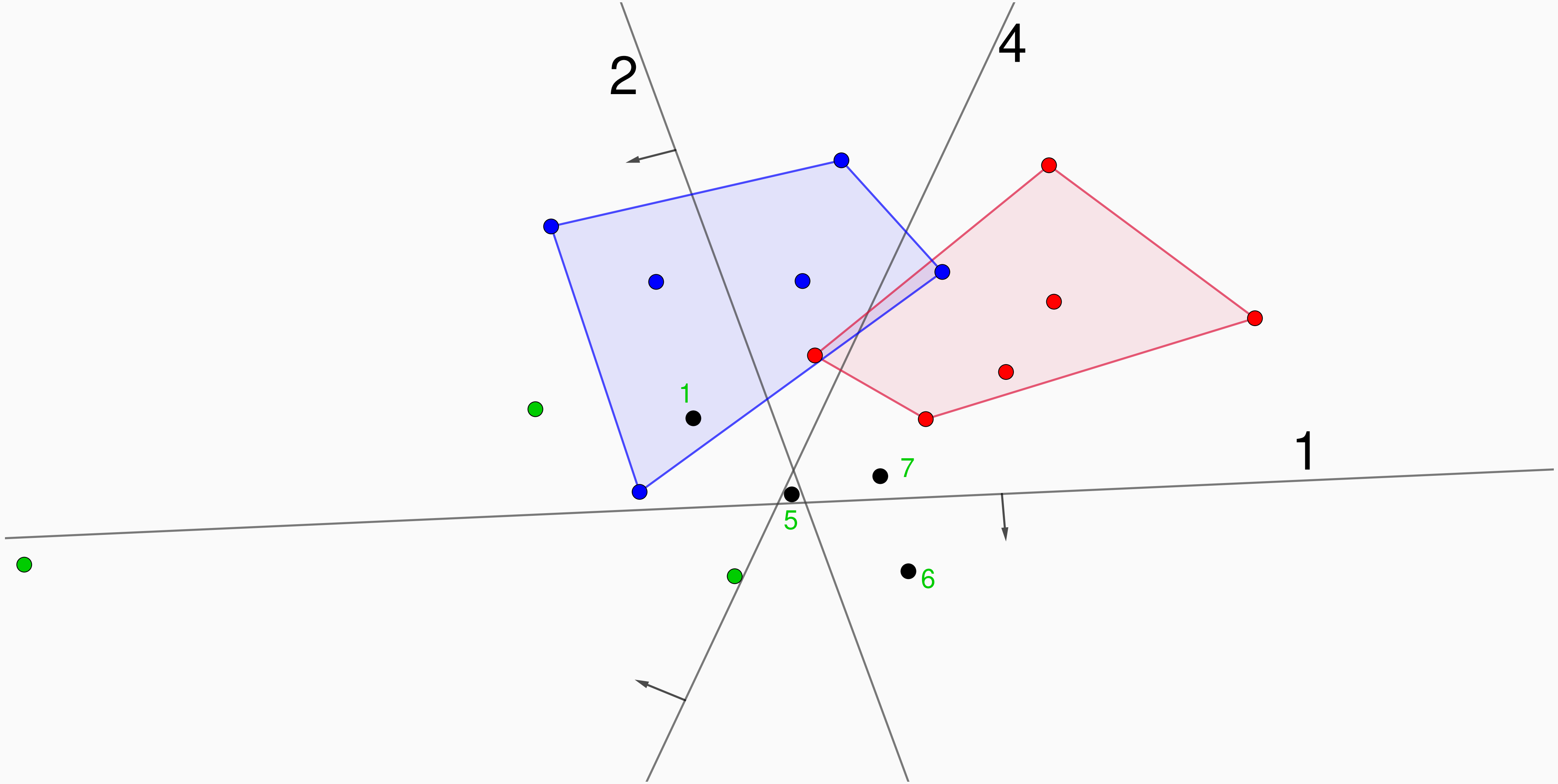
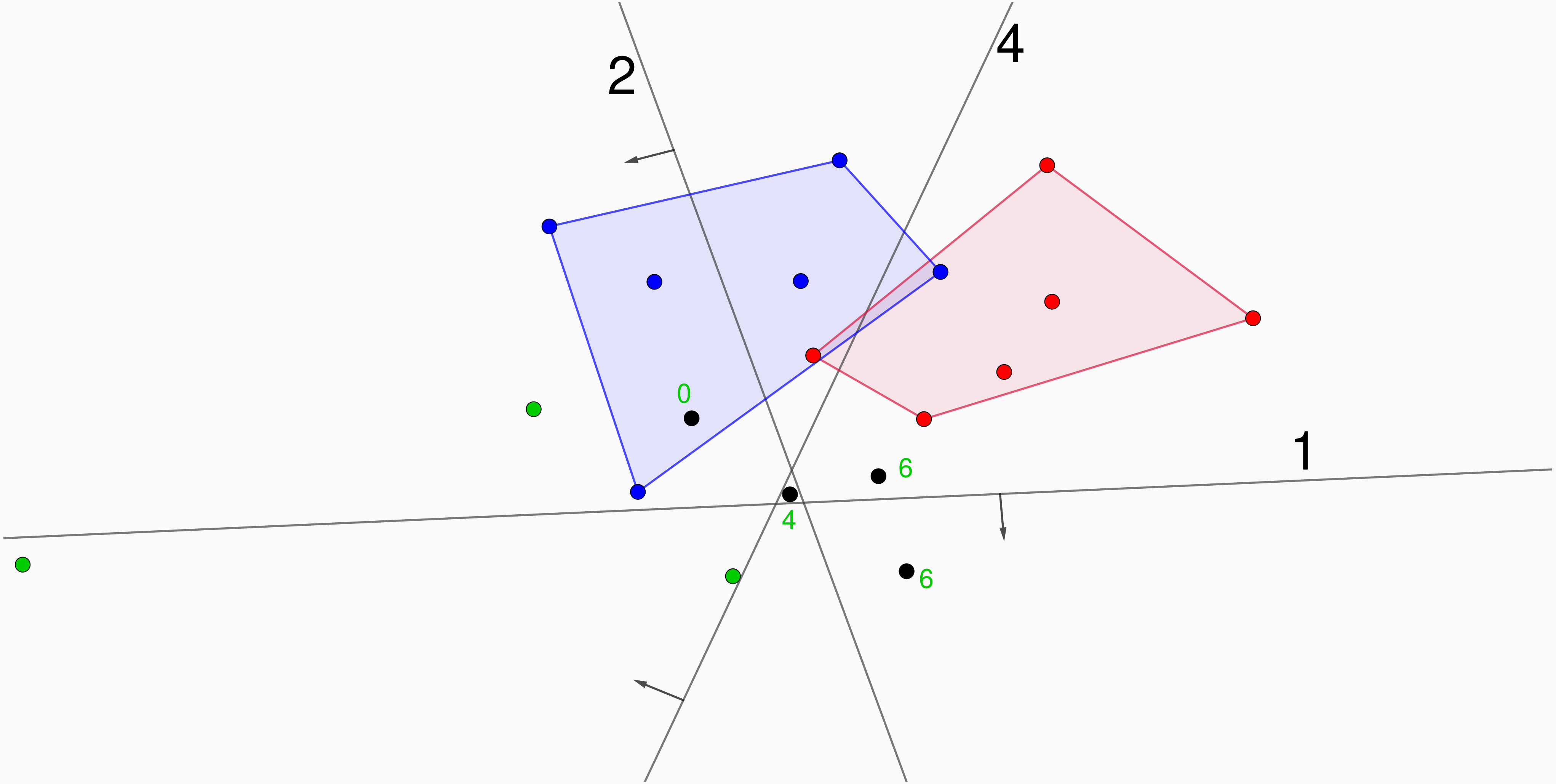



2
4
1
7
5
6
1

A greedy algorithm



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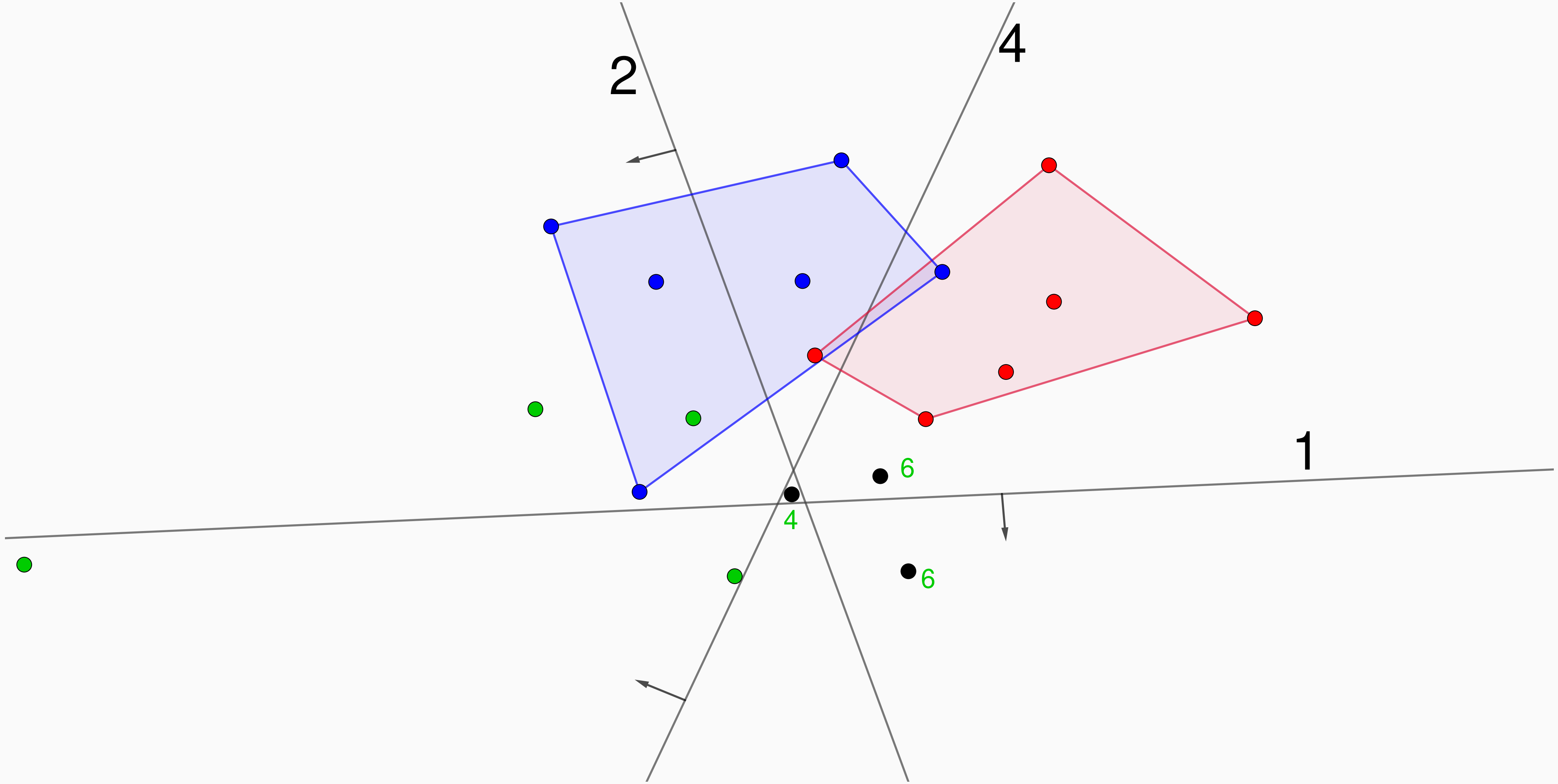

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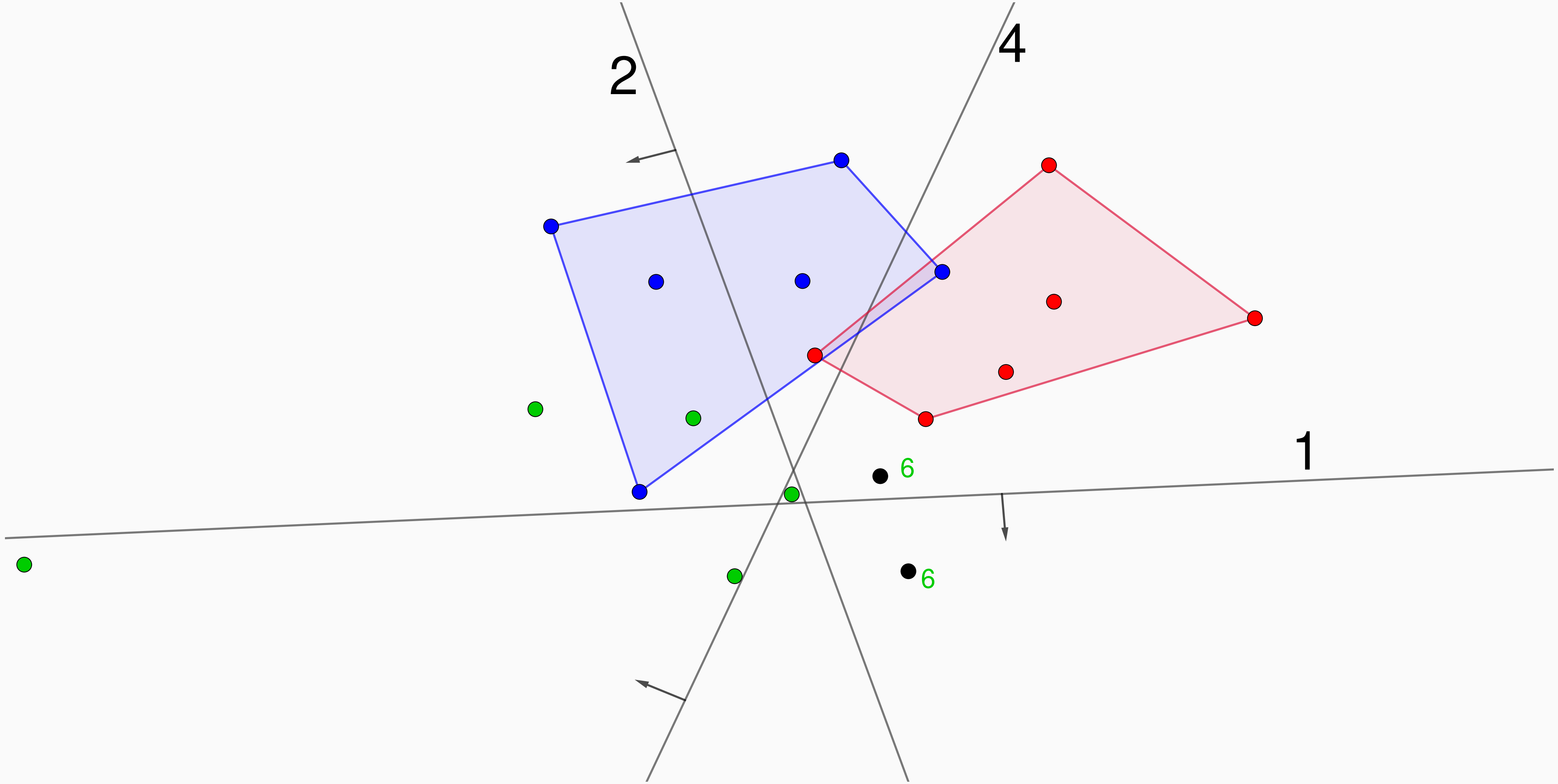

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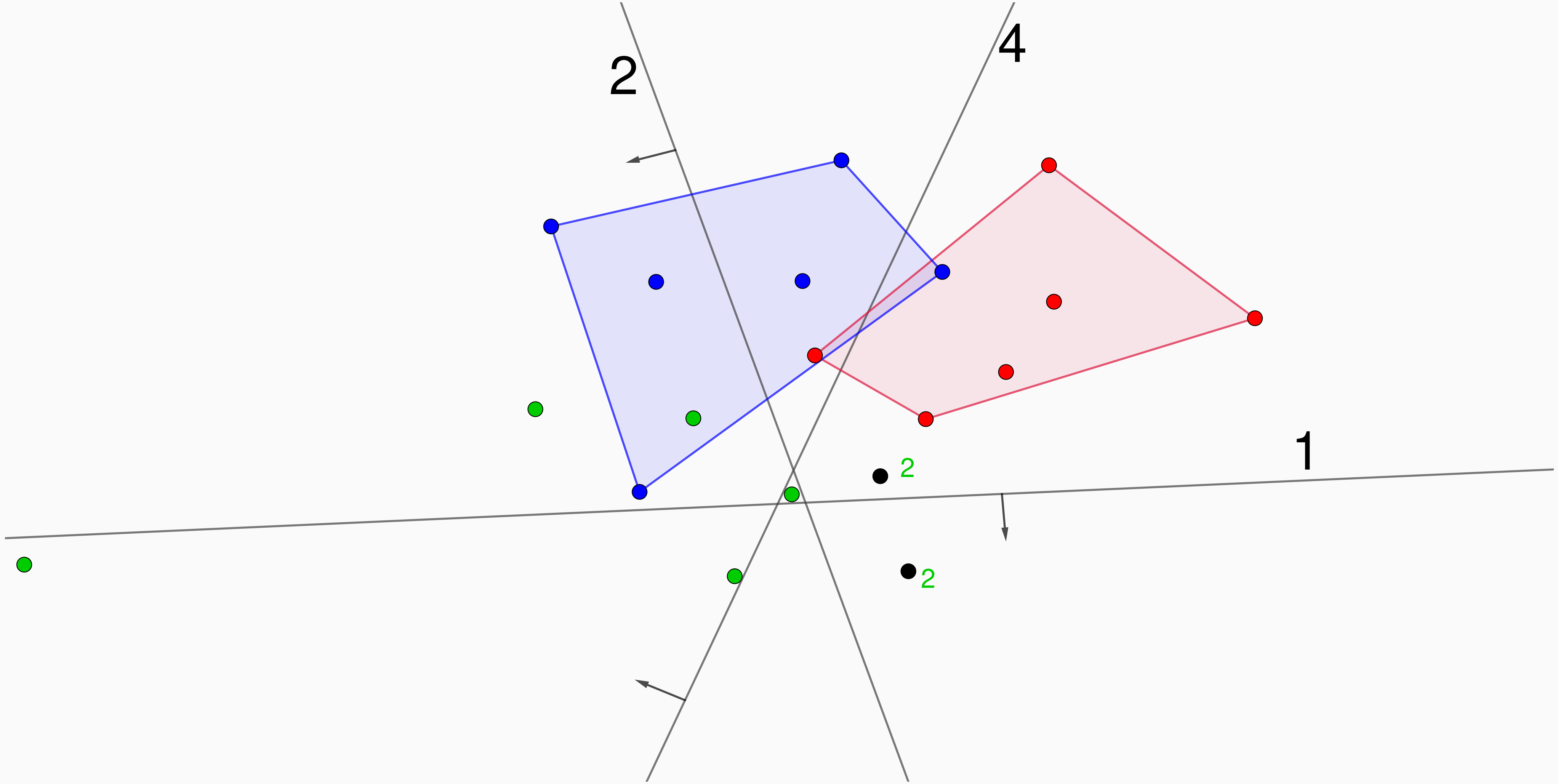

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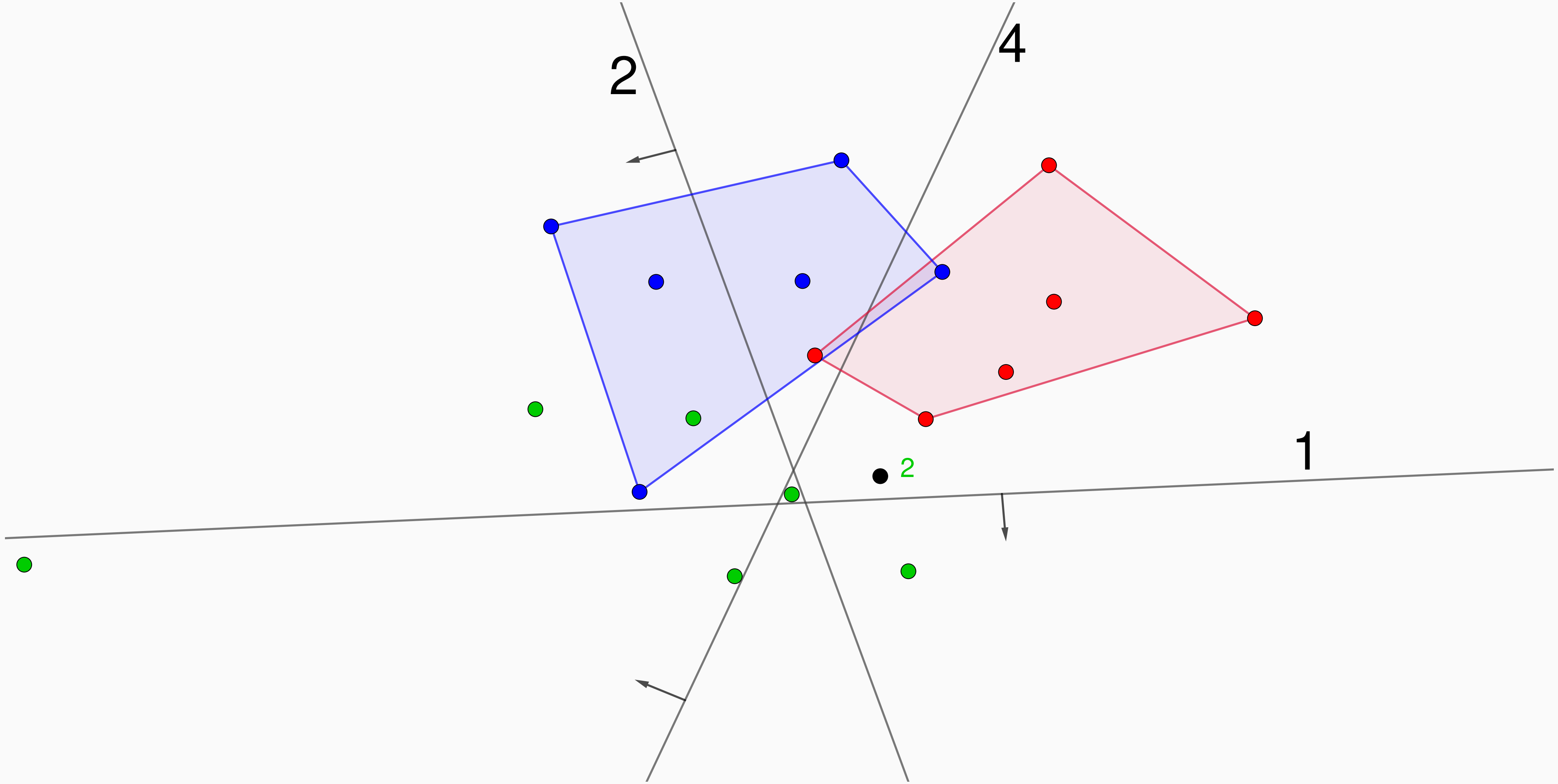

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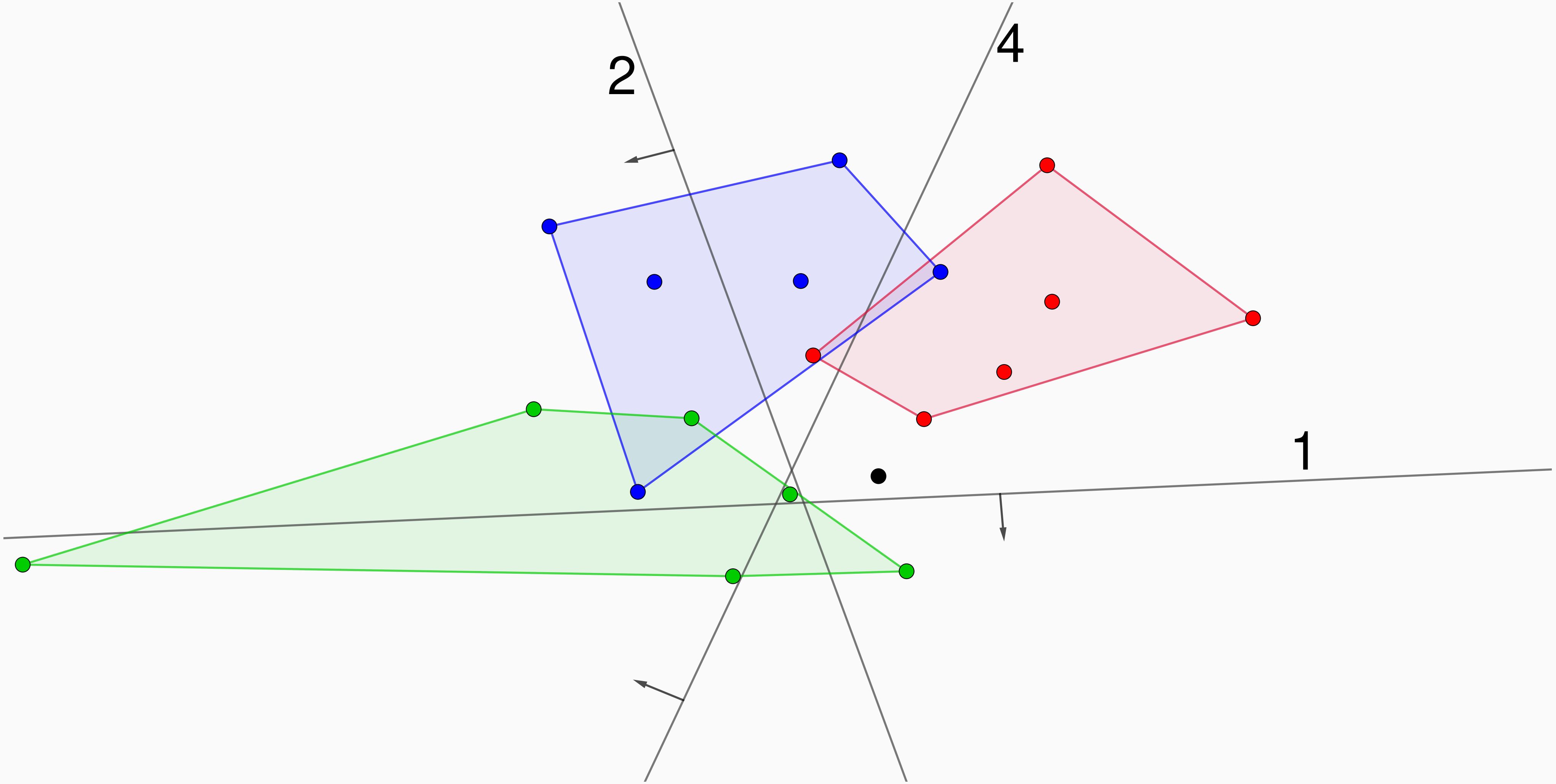

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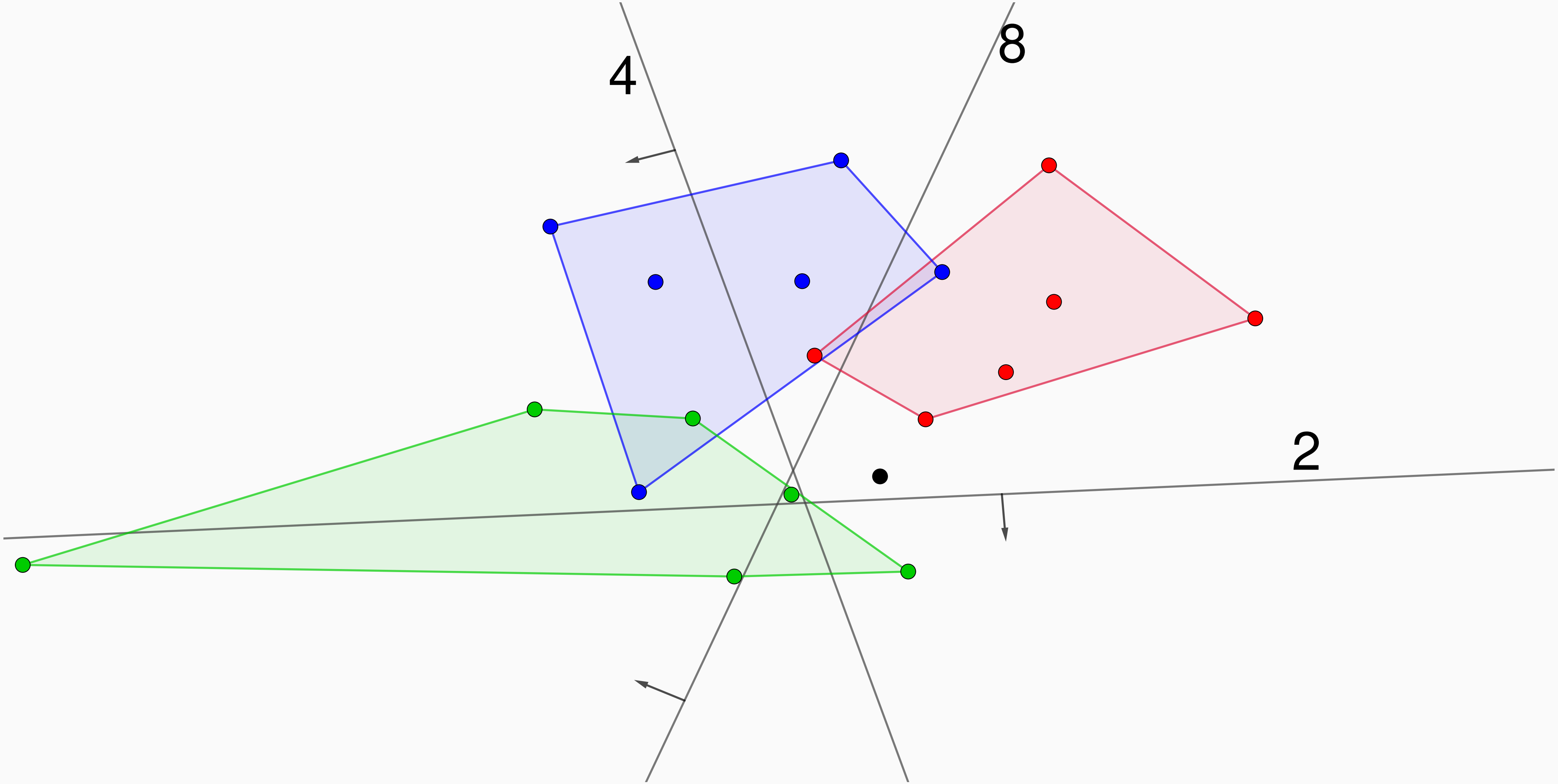
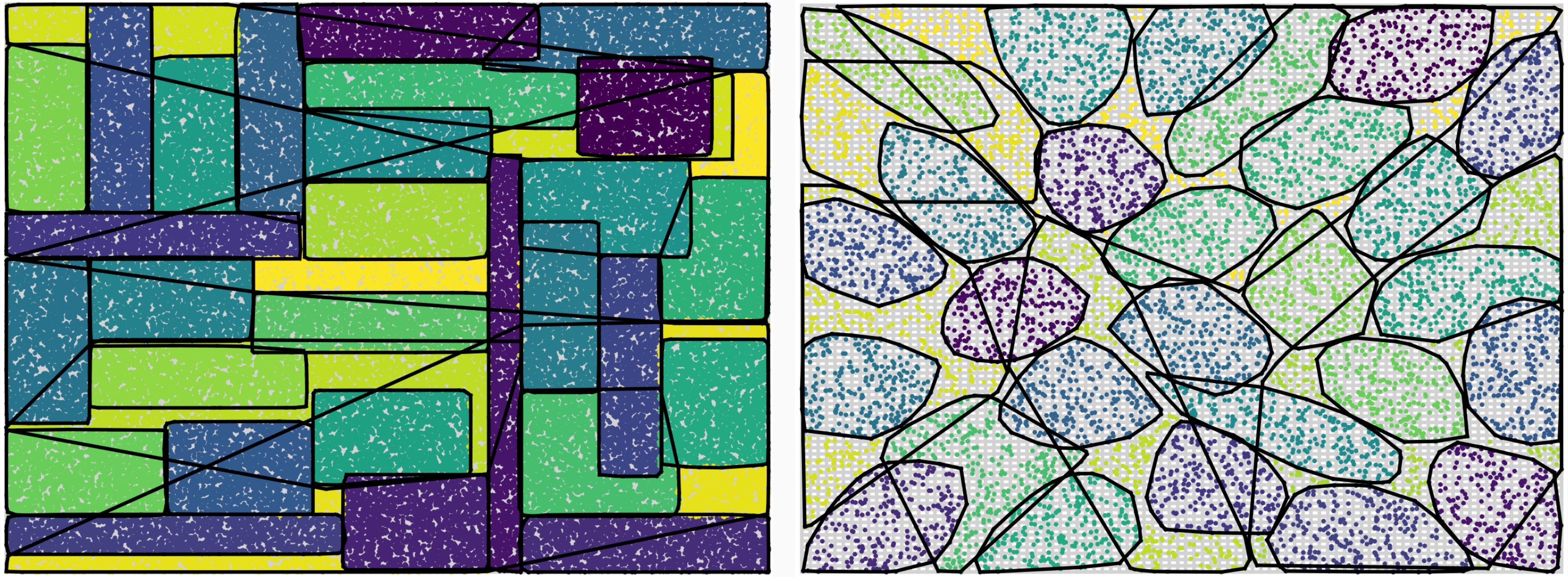


Experiments



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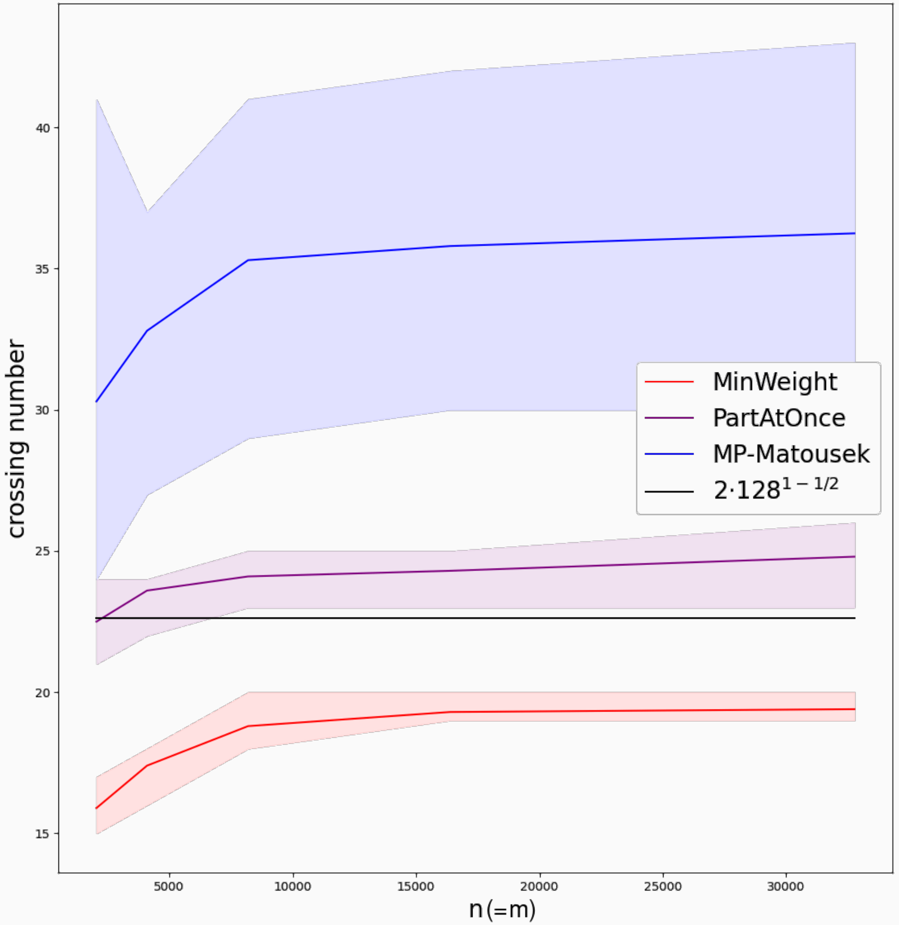

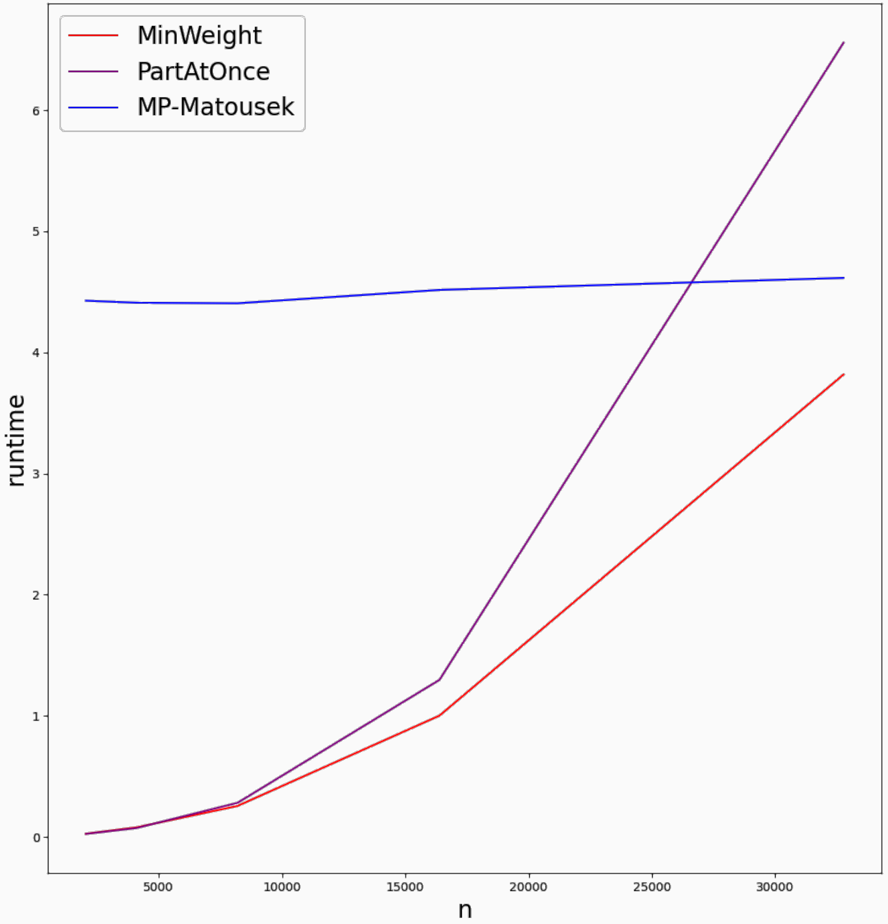

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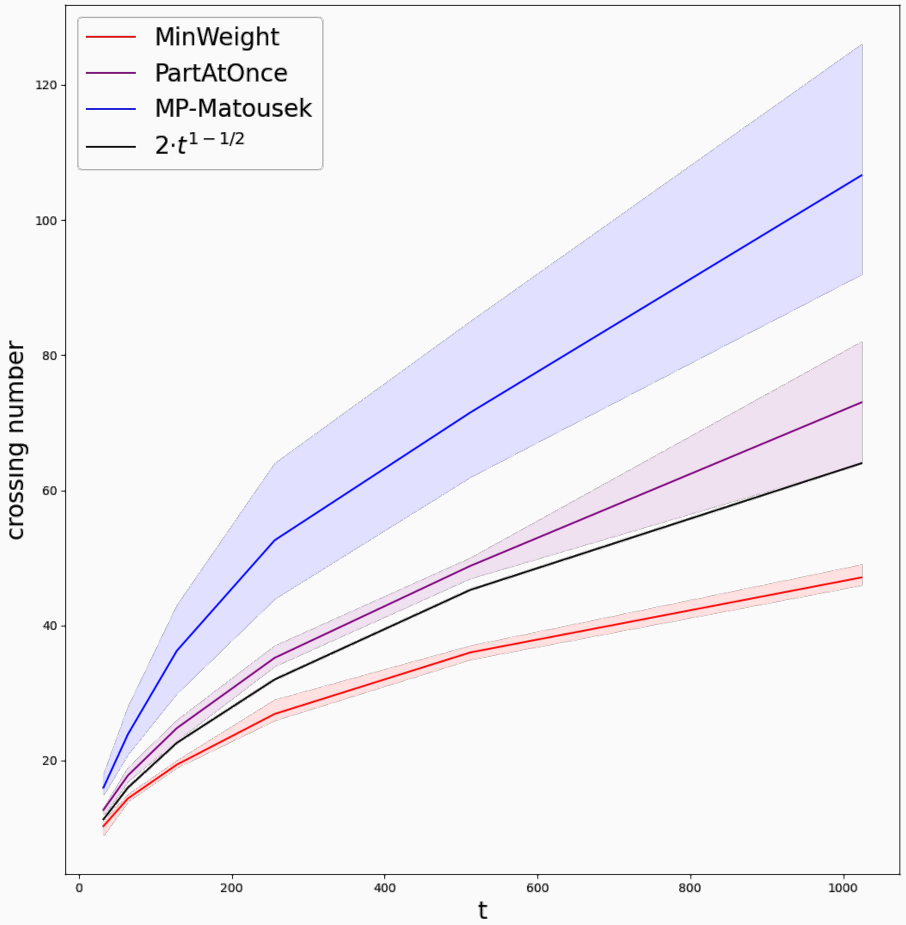

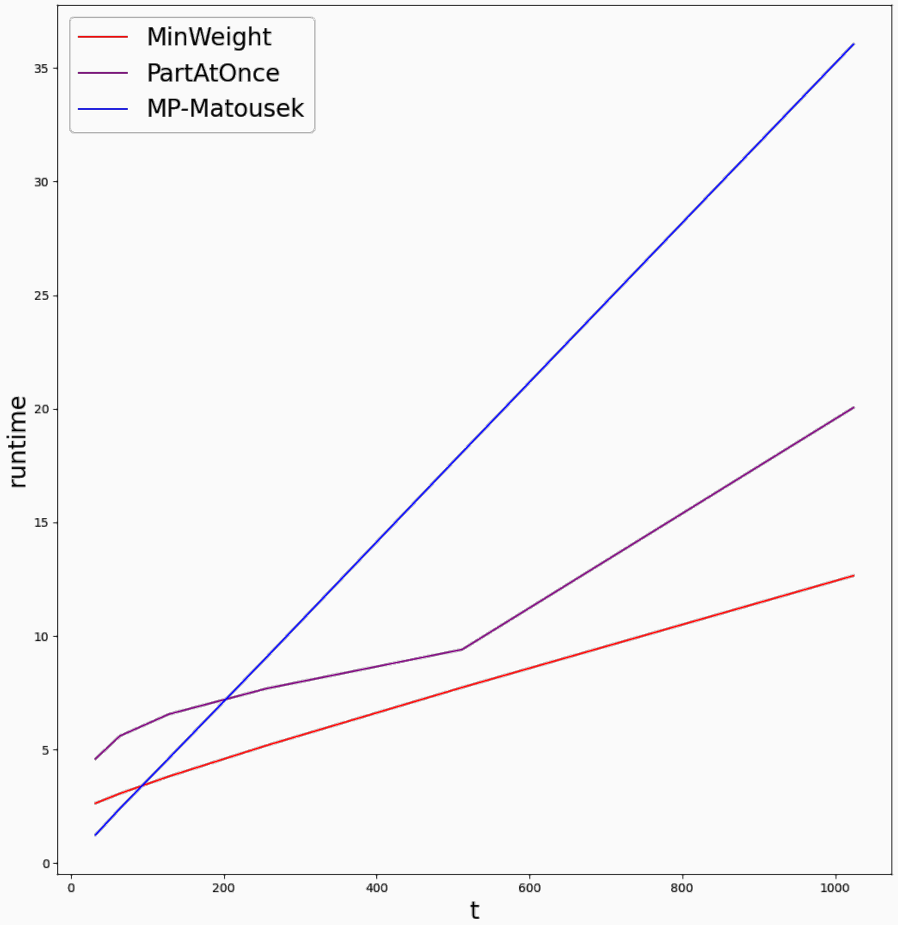

Results on ε-approximations

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Construction from low-crossing partition of size $\varepsilon n \Rightarrow$

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Construction from low-crossing partition of size $\varepsilon n \Rightarrow \frac{1}{\sqrt{(\varepsilon n)^{\frac{d+1}{d}}}}$-approximations w.h.p.

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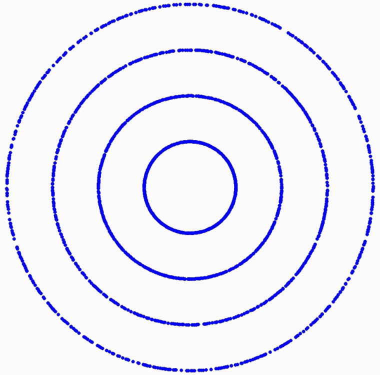
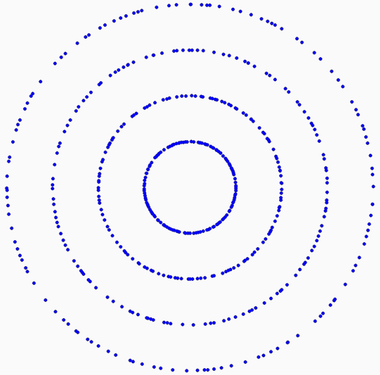
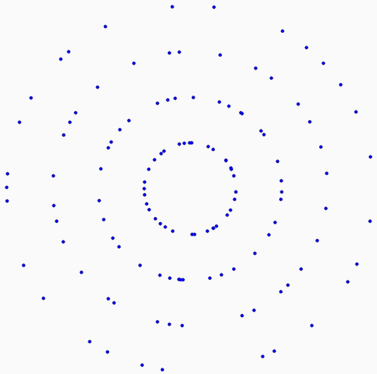
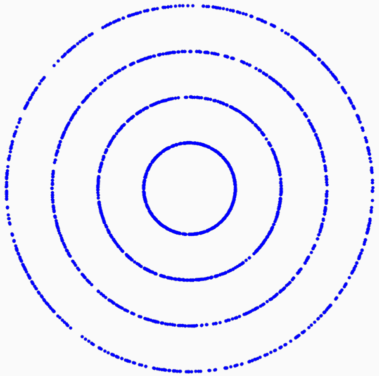
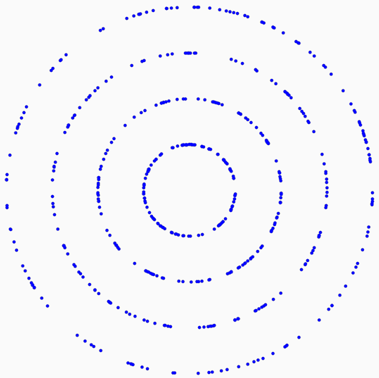
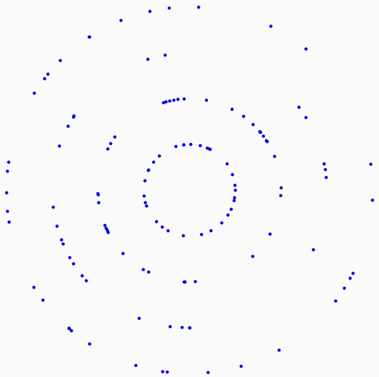
Suri, Toth, Zhou (2006)

Results on ε-approximations

	Approx factor	Sample size: $\frac{n}{4}$	Sample size: $\frac{n}{16}$	Sample size: $\frac{n}{64}$
Our algorithm	$\frac{1}{\sqrt{(\varepsilon n)^{\frac{d+1}{d}}}}$			
A uniform random sample	$\frac{1}{\sqrt{\varepsilon n}}$			

Perspectives

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Thank you

Bibliography

[Ale90] Alexander, Ralph: Geometric methods in the study of irregularities of distribution. In: *Combinatorica* vol. 10, Springer (1990)

[Ban10] Bansal, Nikhil: Constructive algorithms for discrepancy minimization. In: *2010 IEEE 51st Annual Symposium on Foundations of Computer Science*, 2010, pp. 3–10

[Cha93] Chazelle, Bernard: Cutting hyperplanes for divide-and-conquer. In: *Discrete & Computational Geometry* vol. 9 (1993)

[Har00] Har-Peled, Sariel: Constructing Planar Cuttings in Theory and Practice. In: *SIAM J. Comput.* vol. 29 (2000)

[Hau95] Haussler, David: Sphere packing numbers for subsets of the Boolean n-cube with bounded Vapnik-Chervonenkis dimension. In: *Journal of Combinatorial Theory, Series A* vol. 69 (1995)

[LM15] Lovett, Shachar ; Meka, Raghu: Constructive Discrepancy Minimization by Walking on the Edges. In: *SIAM Journal on Computing* vol. 44 (2015)

[LRR17] Levy, Avi ; Ramadas, Harishchandra ; Rothvoss, Thomas: Deterministic Discrepancy Minimization via the Multiplicative Weight Update Method. In: *Integer Programming and Combinatorial Optimization*, 2017

Bibliography

[Mat92] Matoušek, Jiří: Efficient partition trees. In: *Discrete & Computational Geometry* vol. 8 (1992)

[MP21] Matheny, Michael ; Phillips, Jeff M: Approximate Maximum Halfspace Discrepancy. In: *32nd International Symposium on Algorithms and Computation (ISAAC 2021)*, 2021

[Spe85] Spencer, Joel: Six standard deviations suffice. In: *Transactions of the American Mathematical Society* vol. 289 (1985)

[STZ06] Suri, Subhash ; Toth, Csaba D ; Zhou, Yunhong: Range Counting over Multidimensional Data Streams. In: *Discrete & Computational Geometry* vol. 36 (2006)

[Tal94] Talagrand, Michel: Sharper Bounds for Gaussian and Empirical Processes. In: *The Annals of Probability* vol. 22 (1994)